MAE 459 Group Project: Liquid Propellant Rocket

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A group of NC State researchers are designing a liquid rocket engine for missions that require high thrust and orbital insertion through combustion of a liquid propellant. As such, the design of the liquid rocket engine will perform a maneuver for a small spacecraft from the bottom of LEO to GEO which allows it to place a satellite within the final orbit. Key areas of focus were choice of propellant, propellant tank design, feed mechanism, engine cycle, ignition and cooling mechanisms, and combustion chamber design. A plan was also developed to test this conceptual design to provide proof of operation.

I Nomenclature

LEO	=	Low Earth Orbit
$ORSC$	=	Oxidizer-Rich Closed Stage Combustion
km	=	Kilometers
GEO	=	Geostationary Orbit
s	=	Seconds
m_o	=	Total Mass of Liquid Rocket Engine = 400 kg
kg	=	Kilograms
MR	=	Mass Ratio = 0.28
m_p	=	Mass of Liquid Propellant
v_p	=	Velocity of Initial Orbit
v_a	=	Velocity of Final Orbit
μ	=	Gravitational Parameter of Earth = 398600 km^3/s^2
r_1	=	Initial Circular Orbit = 6,538 km
r_2	=	Initial Circular Orbit = 42,164 km
a	=	Semi-major Axis of Transfer Orbit
v_{pt}	=	Velocity at Perigee of Transfer Orbit
v_{at}	=	Velocity at Apogee of Transfer Orbit
Δv_1	=	Velocity Burn at Transfer Perigee
Δv_2	=	Velocity Burn at Transfer Apogee
Δv	=	Total Velocity Burn of Hohmann Transfer
I_s	=	Specific Impulse of Liquid Rocket Engine
g_0	=	Acceleration Due to Gravity = 9.81 m/s^2
ma_{fuel}	=	Atomic Mass of Fuel
$ma_{oxidizer}$	=	Atomic Mass of Oxidizer
ma_C	=	Atomic Mass of Carbon = 12.011 amu
$ma_{Hydrogen}$	=	Atomic Mass of Hydrogen = 1.00784 amu
ma_O	=	Atomic Mass of Oxygen = 15.999 amu
m_{fuel}	=	Mass of Fuel
$m_{oxidizer}$	=	Mass of Oxidizer
ρ_{fuel}	=	Density of Fuel = 440 g/m^3
$\rho_{oxidizer}$	=	Density of Oxidizer = 1141 g/m^3
V_{fuel}	=	Volume of Fuel
$V_{oxidizer}$	=	Volume of Oxidizer
r_{fuel}	=	Radius of Fuel Tank
$r_{oxidizer}$	=	Radius of Oxidizer Tank

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P_c	=	Combustor Chamber Pressure
P_{inj}	=	Injector Pressure
P_a	=	Ambient Pressure of Combustor Chamber
P_t	=	Throat Pressure of Combustor Chamber
$\dot{m}_f$	=	Fuel Mass Flow Rate
$\dot{m}_{ox}$	=	Oxidizer Mass Flow Rate
O/F	=	Propellant Mixture Ratio
ϕ	=	Equivalence Ratio
P_e	=	Exit Pressure of Combustor
T_e	=	Exit Temperature of Combustor
M_e	=	Exit Mach Number of Combustor
V_e	=	Exit Velocity of Combustor
$\frac{A_e}{A_t}$	=	Expansion Ratio of Combustor
A_e	=	Combustor Exit Area
A_t	=	Combustor Throat Area
$\dot{m}$	=	Propellant Mass Flow Rate
c^*	=	Characteristic Velocity
F	=	Combustor Thrust
C_F	=	Thrust Coefficient
T_c	=	Combustor Chamber Temperature
γ	=	Ratio of Specific Heat
R	=	Gas Constant
$\frac{P_e}{P_c}$	=	Pressure Ratio of Combustor
K	=	Kelvin
a	=	Speed of Sound
Pa	=	Pascals
A_c	=	Combustor Chamber Area
V_c	=	Volume of Combustion Chamber
L^*	=	Characteristic Length of Nozzle
L_c	=	Length of Combustion Chamber
r_e	=	Exit Radius of Combustion Chamber
r_t	=	Throat Radius of Combustion Chamber
r_c	=	Radius of Combustion Chamber
L_2	=	Length of Nozzle from Throat to Exit
L_1	=	Length of Nozzle from Chamber to Throat

II Introduction

A team of researchers at NC State ask for the modeling of a liquid rocket engine, modeling both it's combustor and nozzle. This liquid rocket engine will be used in a Hohmann transfer orbit, transferring from a low earth orbit, LEO, of 6538 km from the Earth's surface to a geostationary orbit, GEO, of 42164 km. This is to send a telecommunications satellite into said orbit. The mission shall be conducted along an equatorial orbit. As such, the liquid rocket engine and it's subsystems must be designed accordingly to this transfer.

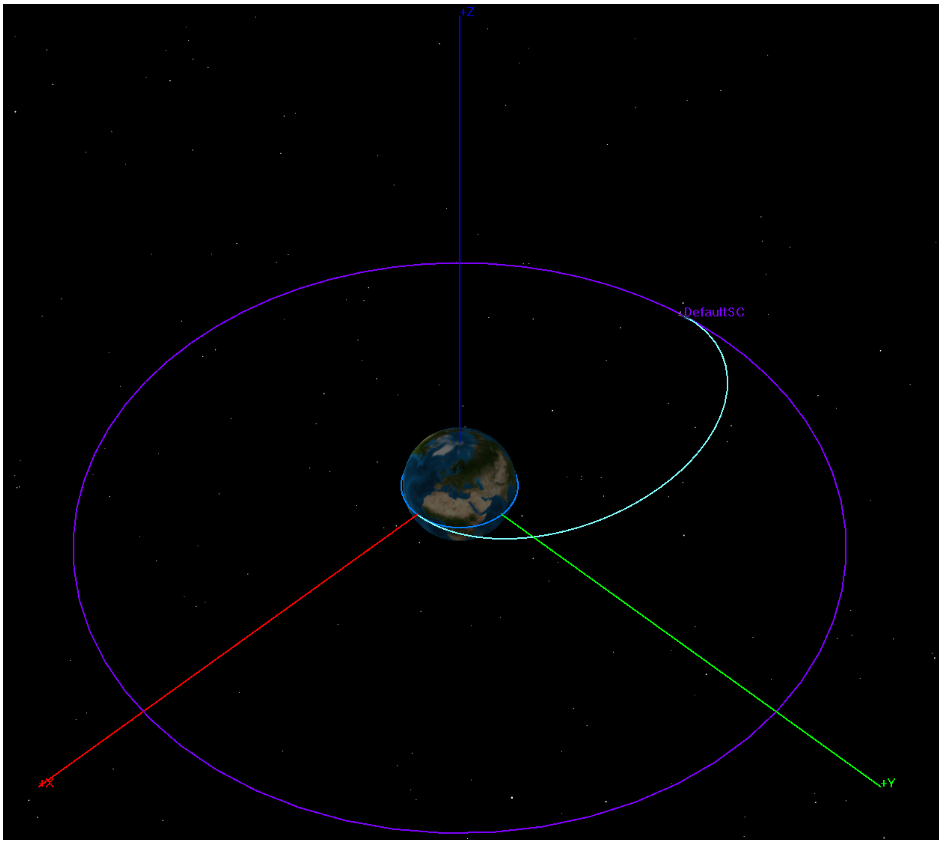

Fig. 1 Mission Orbit

The liquid propellant used will be methalox. It is a mixture of liquid methane the fuel and liquid oxygen shall be used for the oxidizer. With the propellant chosen can the propellant storage tank be decided, figuring out it's material, structure, and dimensions based on the propellant properties. The feed mechanism, tank pressurization, and engine cycle shall also be determined. Alongside those shall the thermal systems for the engine be considered between the ignition and cooling mechanism. These parameters will be chosen on the basis of a variety of analytical and computational fluid dynamics (CFD) models.

III Literature Review

Bipropellant Liquid rocket engines have been an established precedent for complex missions that require high thrust and orbit insertion due to reliable performance across a broad range of operating conditions. These engines work by pumping liquid fuel and oxidizer from separate tanks into a combustion chamber. In this chamber, the fuel and oxidizer mix and burn to create hot, pressurized gas that is expelled at a high velocity through a nozzle. This results in thrust that propels the rocket forward [1]. These types of engines are highly controllable and are the primary propulsion for most space launch vehicles, such as SpaceX's Falcon 9 and Blue Origin's New Shepard [2]. Modern engine development has depended heavily on decades of propellant research and optimization as engine cycles, combustion modeling, and thermal management are consistently characteristics of importance. Given these operational requirements, the choice of propellant becomes a critical factor in determining engine performance and overall system design.

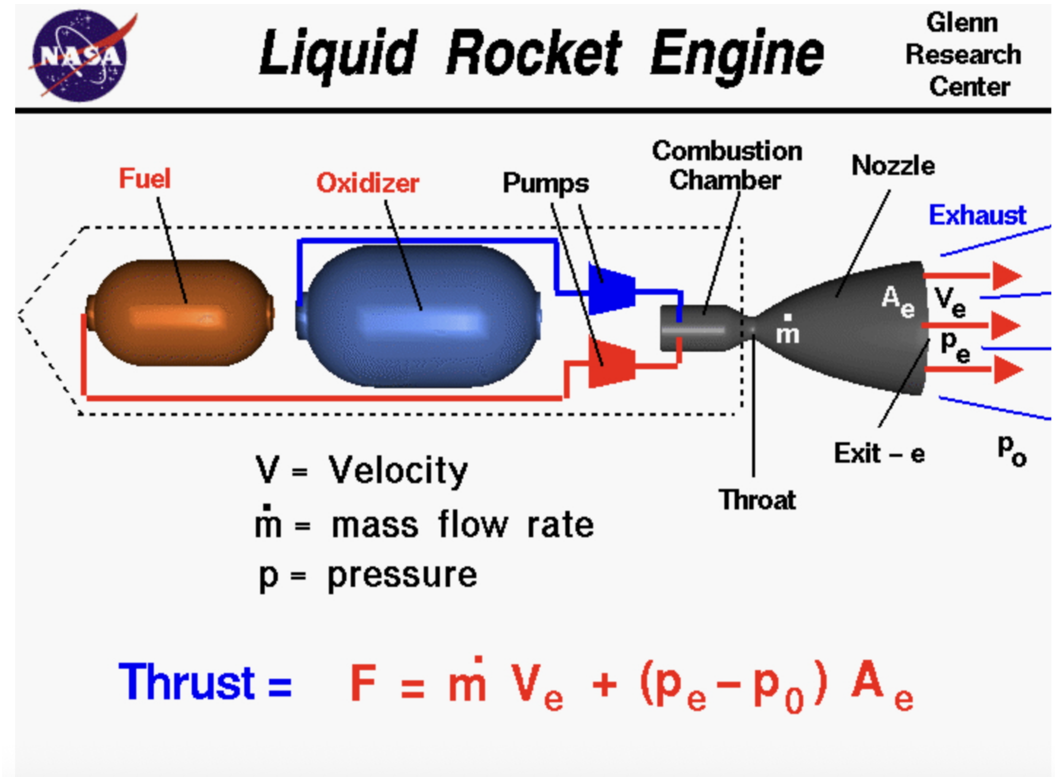

Fig. 2 Breakdown of a Liquid Rocket Engine [3]

Although there are numerous liquid bipropellant combinations, some are more widely used than others including LOX/RP-1, LOX/LH_2, and LOX/LCH_4. LOX/RP-1 is a cryogenic liquid oxygen and refined kerosene mixture that is one of the most historically important bipropellants. This combination has a high thrust capability and density, which allows for smaller tank volumes, easier storage, and lighter structures. However, some disadvantages of this propellant are a moderately low specific impulse and the generation of carbon deposits in cooling channels that can cause coking. Some examples of LOX/RP-1 being used are in early NASA launch vehicles and SpaceX's Falcon 9 Merlin engines [4]. Next, LOX/LH_2 is a liquid oxygen and liquid hydrogen mixture that is extremely light and cryogenic. This bipropellant has the highest specific impulse of any widely used chemical propellant of a value around 450s. It also provides very clean combustion and is ideal for upper stages and deep-space missions. Some disadvantages include having a lower density which requires a larger tank and being more expensive to handle. The most common examples of LOX/LH_2 being used are in NASA's space shuttle main engines and Delta IV RS-68 engines [5]. Lastly, LOX/LCH_4 is a newer bipropellant that is a mixture of liquid oxygen and liquid methane. This combination is easier to store than LH_2, due to its higher density, and it provides cleaner combustion than RP-1. It also offers high performance and has beneficial cooling performance. Some disadvantages include having a lower specific impulse than a hydrogen bipropellant and it is not as historically proven as some other combinations. The most notable examples of this mixture being used are in SpaceX's Raptor and Blue Origin's BE-4 [6]. Due to the nature of the mission, LOX/LCH_4, or methalox, was chosen as the desired bipropellent.

With the propellant combinations established, the next critical design decision is the selection of an appropriate engine cycle. An engine cycle describes how propellants are pressurized, delivered, and combusted within the engine. The most simple approach to this is a pressure-fed system. Pressure-fed engine cycles work by utilizing a separate, high-pressure gas that pushes both the fuel and oxidizer from their tanks directly into the combustion chamber. This is a mechanically simple system that is historically reliable however, it is constrained by upper limits of chamber pressure and is typically only used by small spacecraft thrusters. These performance limitations make them unsuitable for missions that require higher specific impulse or larger delta-v budgets, such as a Hohmann transfer from LEO to GEO [7].

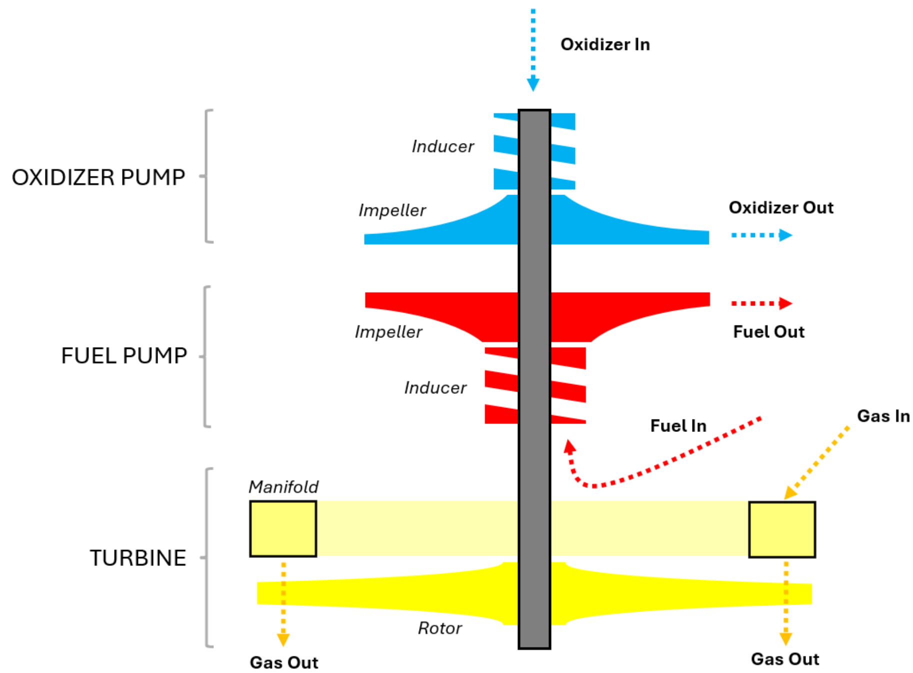

Fig. 3 A Common Turbopump Arrangement [8]

In order to accommodate these requirements, modern launch vehicles tend to rely on pump-fed engine cycles. These types of engine cycles utilize turbopumps to pressurize the propellant before combustion, which provide higher performance with less mass. While several cycle types exist, staged-combustion cycles have emerged as some of the most efficient chemical propulsion systems developed. Staged combustion works by burning fuel and oxidizer in multiple steps, initially in a preburner and then the main chamber. By burning a small portion of the propellant in a preburner, hot gas is produced which powers the turbopump. This gas is then fed into the main chamber to complete combustion without wasting any propellant [9]. The specific variant that will be utilized for this mission is an oxidizer-rich closed stage combustion (ORSC) cycle. In these systems, the preburner operates fuel-lean and the hot gas that drives the turbines before entering the main combustion chamber is oxygen-rich. This supports a high chamber pressure and better overall propellant utilization. Historically, ORSC cycles have been implemented within Russian engines such as the RD-170 and RD-180. The RD-170 is the most powerful liquid-rocket engine ever flown as it provided over 1.6 million pounds of thrust. The RD-180 built off of the RD-170 and was used successfully as the first-stage engine for the Atlas V [10]. This implementation demonstrated the reliability of ORSC cycles in modern launch vehicles. Currently, this cycle has been adopted in methane engines like Blue Origin's BE-4, which makes it the first large-scale American ORSC methalox engine, demonstrating the feasibility for these systems for missions requiring higher delta-v, such as LEO-to-GEO transfers.

Ignition and cooling mechanisms are responsible for reliable startup and sustained operation for liquid rocket engines. Currently, the state-of-the-art for booster-class engines is torch ignition systems, as they provide reliable lighting under cryogenic flow conditions. This system works by releasing pressurized fuel through a nozzle and, right at the nozzle's exit, a spark plug ignites the mixture. This produces a focused, high temperature flame that then impinges on the main engine's propellant injectors. This method has become the preferred ignition approach for modern engines because of its ability to provide stable, repeatable starts while also minimizing the risk of hard-start events at ignition [11].

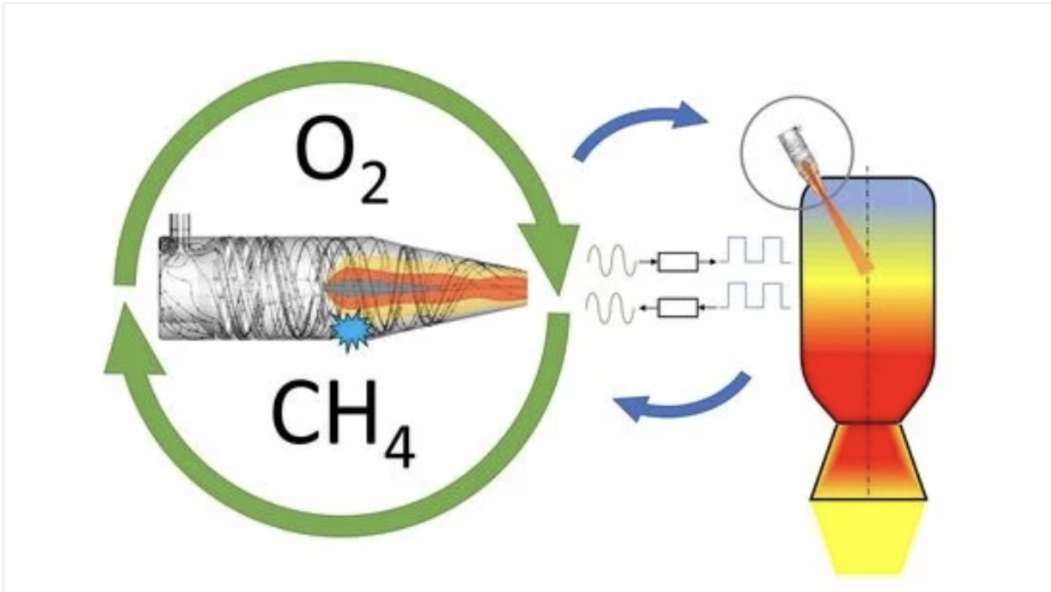

Fig. 4 Breakdown of a Torch Ignition System for Methalox Bipropellant [11]

Once ignition is achieved, the engine must withstand the extreme thermal loads that come with sustaining operation. In order to do so, a cooling mechanism is necessary. A widely adopted method is regenerative cooling, which is the cooling mechanism that will be utilized in our mission. Regenerative cooling uses a cryogenic propellant that circulates throughout the chamber and nozzle walls before entering the injector. Some advantages of this system is that it absorbs heat efficiently, prevents structural failure, and improves combustion performance. There are also some techniques that can be used in addition to regenerative cooling such as film cooling or ablative liners. These are useful for engines with extremely chamber temperatures or transient thermal loadings [12].

IV Methodology/Calculations

IV.I Mass Ratios and Propellant Mass

A liquid rocket engine with a total mass, m_o, of 400 kg shall be sufficient to carry out the desired mission. Given the payload being a telecomunications satellite, the mass ratio, MR, is 0.33. That leaves the propellant mass ratio as 0.67. With these parameters can the total propellant mass, m_p, be calculated. This can be done simply by multiplying the propellant mass ratio by the mass of the liquid rocket engine. Doing so leads the propellant mass to weigh 260 kg.

$$m_p = (1 - MR) * m_o = 268 \text{ kg}$$

IV.II Hohmann Transfer Parameters

Before a proper propellant can be selected, the mission requirements must be detailed as a means to choose the best propellant. As stated before, the rocket will start at a LEO of 6538 km, flying at a velocity of 7.8081 km/s. For the sake of this mission, that is the velocity at 'perigee' for this mission or the initial orbital velocity, v_p. The rocket will be flying to a GEO of 42164 km, it's orbital velocity reaching 3.0747 km/s. This will be it's velocity at 'apogee' or the final orbital velocity, v_a. Since these are circular orbits, these are calculated simply by the square root of the gravitational parameter of Earth, μ, divided by the orbital radius.

$$v_p = \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{3.986 \times 10^5}{6378 + 160}} = 7.8081 \text{ km/s}$$

$$v_a = \sqrt{\frac{\mu}{r_2}} = \sqrt{\frac{3.986 \times 10^5}{6378 + 35786}} = 3.0747 \text{ km/s}$$

Then comes determining the Hohmann transfer itself. First is the semi-major axis of the transfer orbit, a, done simply by taking the sum of the two orbits and taking their average. Then does one need to find the velocity at the transfer perigee ,v_{pt}, I.E. the velocity gained after entering the transfer orbit. Additionally, the velocity at the transfer apogee, v_{at}, I.E. the velocity gained before leaving the transfer orbit needs to be found.

$$a = \frac{r_1 + r_2}{2} = 24351$$

$$v_{pt} = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a}\right)} = 10.2745 \text{ km/s}$$

$$v_{at} = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a}\right)} = 1.5932 \text{ km/s}$$

With the parameters of the transfer orbit found can the required velocity burns be found. The burn at the transfer perigee, Δv_1, can be found by subtracting the velocity at the transfer perigee by the initial orbit velocity. The burn at the transfer apogee, Δv_2, can be found by subtracting the final orbit velocity by the transfer apogee velocity. Given both values can one find the total velocity burn needed for the mission, Δv, done simply by adding the absolute values of the aforementioned velocity burns. As such, assuming an equatorial orbit, the total velocity burn needed for this mission is 3.9478 km/s.

$$\Delta v_1 = v_{pt} - v_p = 2.4663 \text{ km/s}$$

$$\Delta v_2 = v_a - v_{at} = 1.4815 \text{ km/s}$$

$$\Delta v = |\Delta v_1| + |\Delta v_2| = 3.9478 \text{ km/s}$$

The velocity burn is used to find the specific impulse, I_s, required for the flight. Given the velocity burn, the acceleration due to Earth's gravity, g_0, and the inverse of the mass ratio can the specific impulse be found. Note that specific impulse should be in seconds, so adjust units accordingly. In this case, the equation was multiplied by one-thousand to get the proper results. As such, the specific impulse of the liquid rocket engine is 316.1354 seconds.

$$I_s = \frac{\Delta v}{g_0 log(\frac{1}{MR})} = 362.9865 \text{ s}$$

IV.III Propellant Storage Tank

Before deciding upon the propellant storage, one needs to analyze the properties of said propellant. Based on the mixing ratio of 3.6, the mass of the oxidizer is 3.6 times greater then that of the fuel. Knowing this value, in conjuncture with the mass of propellant from the mass ratio, the mass of the fuel and oxidizer can be calculated as shown below.

$$m_{fuel} = m_p/4.6 = 58.2609 \text{ kg}$$

$$m_{oxidizer} = 3.6 * m_{fuel} = 209.7391 \text{ kg}$$

With the density of the fuel, ρ_{fuel}, being 440 kg/m^3 and the density of the oxidizer being $\rho_{oxidizer}$, being 1141 kg/m^3 can the volume of both be found. This is done simply by dividing the mass by the density. As such, the fuel volume, V_{fuel}, is 0.1324 m^3 and the oxidizer volume, $V_{oxidizer}$, is 0.1838 m^3.

$$V_{fuel} = \frac{m_{fuel}}{\rho_{fuel}} = 0.1324 \text{ m}^3$$

$$V_{oxidizer} = \frac{m_{oxidizer}}{\rho_{oxidizer}} = 0.1838 \text{ m}^3$$

Using this volume, it is possible to determine the geometry of the tanks. Given that this stage is operating only in orbit, and the need for high strength within the tank due to its internal pressure, compared to weight, a spherical shape was chosen. This offers the highest strength to weight ratio of any geometric configuration and generally requires less room within the rocket. This comes at the tradeoff of being more challenging to machine. With the volume found the radius of the propellant tanks can be calculated. The radius of the fuel tanks, r_{fuel} calculates to be 0.3162 m while the radius of the oxidizer tanks, $r_{oxidizer}$, are 0.3527 m. In reality, an extra 5% would be added to the tanks volume to account for any vaporization resulting in pressure increases.

$$r_{fuel} = (\frac{V_{fuel}}{\frac{4}{3}\pi})^{\frac{1}{3}} = 0.3162 \text{ m}$$

$$r_{oxidizer} = (\frac{V_{oxidizer}}{\frac{4}{3}\pi})^{\frac{1}{3}} = 0.3527 \text{ m}$$

IV.IV Propellant Stoichiometry

To be able to make calculations about the Combustion Chamber and Nozzle parameters, we first need to be able to determine the stoichiometric data from the fuel we selected. To determine these values, we used NASA's CEArun program which can be found at this link. This program calculates various parameters that derive from the propellant we chose and boundary conditions we set. The program has 7 parameters that we can change in order to get values that are listed in order below.

- **Problem Type**
  - Must select problem type as a Rocket problem
- **Pressure**
  - A range of values can be chosen to see how chamber pressure (P_c) affects the performance parameters of the nozzle and chamber.
  - We chose a bounds of 30 bar to 100 bar[13] for the performance of our rocket. The bounds were determined by looking at the performance of rockets today and having pressures similar to those.
- **Fuels**
  - The selected fuel type was set as CH_4 (L).
  - We made an assumption that the liquid methane would be kept at a temperature of 110[14] degrees Kelvin as that is close to the fuel's boiling point and a common temperature used to store it.
- **Oxidizer**
  - The selected oxidizer was O_2 (L).
  - We made the assumption that the liquid oxygen would be kept at a temperature of 90 degrees kelvin. 90 kelvin is around the boiling temperature for te oxidizer and is a common temperature used to store it.
- **Oxidizer/Fuel Ratio**
  - A range of mixture ratios could be selected to see how that affects our performance parameters.
  - The bounds for the values chosen for our mixture ratio comes from the engine cycle we selected and have written about below. Since the engine cycle we chose is oxidizer rich, the bounds of the O/F ratio chosen was 3.4-3.8.
- **Exit Conditions**
  - The exit condition is an optional parameter that can be selected and is very important to help us determine the stoichiometric condition we select for our fuel and oxidizer.
  - The parameters we chose to adjust is the Supersonic Area Ratio as the data from the program would give us the calculated I_{sp} for that area ratio
  - We would then set a range of area ratios and see if our calculated required I_{sp} value would fall in between two calculated I_{sp} values
- **Final Parameters**
  - The last parameters we could adjust were miscellaneous settings for achieving more accurate calculations.
  - We set the combustion chamber to be a finite area and assumed the contraction Ratio to be 8[15] as this is common for smaller sized rockets.
  - Finally the rockets propellant was selected to be in equilibrium which means that the program would be calculating best case performance.

After inputting all of the parameters, the CEArun program would output a lot of variables that we would be able to compare. The three most important parameters we would look for is a higher O/F ratio, a good chamber pressure reading based off of previous combustion chamber design, and finally that it would meet our I_{sp} requirement of 362.9865 seconds or 3560.897 m/s as it would show on the data sheet. Below will be the isentropic conditions we selected.

The NASA CEA program provided the baseline performance parameters for this methalox engine configuration, giving a chamber pressure of approximately 90 bar, an oxidizer to fuel ratio of 3.6, and an expansion ratio A_e/A_t of

Reactant	Wt Fraction	Energy (kJ/kg-mol)	Temp (K)
Fuel: CH_4(L)	1.0000000	-89233.000	111.643
Oxidizer: O_2(L)	1.0000000	-12979.000	90.000
O/F = 3.60000, %Fuel = 21.739130, Eq. Ratio = 1.108129, ϕ = 1.108129			

	Chamber	Throat	Exit 1	Exit 2	Exit 3	Exit 4	Exit 5
P_∞/P	1.0000	1.7263	811.34	880.16	978.37	992.57	1006.81
P (bar)	90.000	52.136	0.11093	0.10225	0.09199	0.09067	0.08939
T (K)	3598.04	3426.58	1779.50	1756.46	1726.83	1722.82	1718.86
ρ (kg/m^3)	6.6977e0	4.1281e0	1.8439e-2	1.7220e-2	1.5758e-2	1.5569e-2	1.5384e-2
H (kJ/kg)	-1526.63	-2237.88	-7864.46	-7913.12	-7975.41	-7983.81	-7992.09
U (kJ/kg)	-2870.38	-3500.83	-8466.06	-8506.92	-8559.16	-8566.21	-8573.15
G (kJ/kg)	-44307.6	-42980.3	-29022.8	-28797.6	-28507.6	-28468.3	-28429.5
S (kJ/kg-K)	11.8901	11.8901	11.8901	11.8901	11.8901	11.8901	11.8901
M (1/n)	22.263	22.559	24.594	24.594	24.595	24.595	24.596
$(dLV/dLP)_T$	-1.04087	-1.03693	-1.00010	-1.00008	-1.00006	-1.00006	-1.00006
$(dLV/dLT)_P$	1.7221	1.6871	1.0034	1.0028	1.0022	1.0022	1.0021
C_p (kJ/kg-K)	7.1018	7.0355	2.1249	2.1137	2.1007	2.0991	2.0975
γ (Cp/Cv)	1.1300	1.1264	1.1906	1.1916	1.1927	1.1929	1.1930
Sound Speed (m/s)	1232.3	1192.7	846.3	841.2	834.4	833.5	832.6
Mach Number	0.000	1.000	4.207	4.249	4.304	4.312	4.319

	Throat	Exit 1	Exit 2	Exit 3	Exit 4	Exit 5
A_e/A_t	1.0000	75.000	80.000	87.000	88.000	89.000
C^* (m/s)	1828.0	1828.0	1828.0	1828.0	1828.0	1828.0
C_f	0.6525	1.9477	1.9552	1.9647	1.9659	1.9672
I_{vac} (m/s)	2251.6	3729.3	3740.1	3753.9	3755.7	3757.5
I_{sp} (m/s)	1192.7	3560.3	3573.9	3591.3	3593.7	3596.0

about 75 for our required I_{sp}. These values are generated under perfect equilibrium conditions in which the chemistry adjusts continuously during expansion. In real engines the flow expands too quickly for full chemical adjustment, so the frozen condition results shown below offer a more realistic lower bound. Actual rocket performance will fall somewhere between the ideal equilibrium predictions and the frozen flow values. In the next section, we will show the frozen parameter conditions and base our nozzle properties off of them.

IV.V Combustion Chamber Dimensions and Calculations

Though NASA's CEArun program found much of these values already, running them through Matlab provides for some redundancy in the calculations while also providing for some values not present in the CEArun program. Even so, some variables found in the CEArun program made for much more exact calculations given specific variables that wouldn't be found otherwise. These include the chamber temperature, T_c, being 3598.04 K, the ratio of specific heats, γ, equaling 1.13 for the chamber, and the characteristic gas constant, R, equaling 373.4655 J/kg-k.

Given the above findings, here are the respective calculations and results for the combustion chamber.

Multiplying the specific impulse by the acceleration due to gravity yields 0.3527 m/s for the exit velocity of the combustion chamber, v_e.

$$v_e = I_s g_o = 0.3527 \text{ m/s}$$

Through the equation below was the pressure ratio, P_e/P_c, found to be 0.0011.

$$\frac{P_e}{P_c} = (1 - \frac{v_e^2(\gamma - 1)}{2\gamma R T_c})^{(\frac{1}{\frac{\gamma-1}{\gamma}})} = 0.0011$$

Using that value, the ratio of specific heats, and the chamber temperature can one find the temperature at the combustor chamber exit, T_e. This is found to be 1645 K.

$$T_e = \frac{T_c}{(\frac{1}{\frac{P_e}{P_c}})^{(\frac{\gamma-1}{\gamma})}} = 1645 \text{ K}$$

Taking the square root of the exit temperature, gas constant, and ratio of specific heats yields the speed of sound, a, at the nozzle's exit. This is 833.2054 m/s. Taking the exit velocity over that value can one additionally find the exit mach number, M_e. This turns out to be supersonic at 4.2737.

$$a = \sqrt{\gamma R T_e} = 833.2054 \text{ m/s}$$

$$M_e = \frac{v_e}{a} = 4.2737$$

Utilizing that exit mach number and the ratio of specific heats in the equation below can the expansion ratio, A_e/A_t, be found. This is the measurement of how much the nozzle expands from the throat. This ratio is found to be 85.0405. Note the reason as to why this value is different from when when using the NASA software is that the software assumes that the fuel is in equilibrium throughout the whole combustion process, whereas these calculations do not. As such, the true expansion ratio lies between the range of 75 to 85.

$$\frac{A_e}{A_t} = \frac{1}{M_e}\sqrt{\left(\frac{1 + 0.5(\gamma - 1)M_e^2}{1 + 0.5(\gamma + 1)}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}} = 85.0405$$

Before moving on to the area of the combustion chamber, the exit pressure for the combustion chamber, P_e, still needs to be found. This is done simply by multiplying the pressure ratio by the chamber pressure. As such, the exit pressure turns out to be 9995.6 Pa.

$$P_e = \left(\frac{P_e}{P_c}\right)P_c = 9995.6 \text{ Pa}$$

Using the exit pressure value just gleaned, the gas constant, exit temperature, exit velocity, and the propellant's mass flow rate can the combustion chamber's exit area, A_e, be found. This equation is a derivation of the ideal gas law, PV= nRT. For the sake of this mission, the total mass flow rate is 1.5 kg/sec. Because of that, the exit area turns out to be 0.0259 m^2. Using that value and dividing it by the calculated expansion ratio yields the area at the throat, A_t. The throat area as such ends up being 0.00030446 m^2.

$$A_e = \frac{\dot{m}}{v_e}\left(\frac{RT_e}{P_e}\right) = 0.0259 \text{ m}^2$$

$$A_t = \frac{A_e}{\frac{A_e}{A_t}} = 0.00030446 \text{ m}^2$$

For the chamber area, multiplying the throat area by the contraction rate equals 0.0024 m^2. For the case of this experiment, the contraction rate is assumed to be 8 to make for the ideal combustion chamber volume.

$$A_c = A_t * 8 = 0.0024 \text{ m}^2$$

For the chamber volume, multiplying the throat area by the characteristic length, L^*, of the nozzle equals 0.00030446 m^3. For the sake of this experiment, the characteristic length is made out as 1 m to make for the ideal combustion chamber volume.

$$V_c = A_t * L^* = 0.00030446 \text{ m}^3$$

Dividing the chamber volume by the chamber area yields a combustion chamber length of 0.125 m.

$$L_c = \frac{V_c}{A_c} = 0.125 \text{ m}$$

Taking the square root of the combustion chamber area over pi yields a exit radius of 0.0908 m. Likewise, doing the same using the different areas for both the throat radius, r_t, and chamber radius, r_c, yields 0.0098 m and 0.0278 m, respectively.

$$r_e = \sqrt{\frac{A_e}{\pi}} = 0.0908 \text{ m}$$

$$r_t = \sqrt{\frac{A_t}{\pi}} = 0.0098 \text{ m}$$

$$r_c = \sqrt{\frac{A_c}{\pi}} = 0.0278 \text{ m}$$

Because of the difference in values caused by the differing expansion ratio in the software and the calculations, the exit radius lies between 0.08 m and 0.0908 m. Additionally, the chamber radius lies between 0.02 m and 0.0278 m. The biggest difference lies in the throat, being within the range of 0.0098 m and 0.04 m. For the sake of simplicity and the potential avoidance of human error, the values found using the NASA software will be used. As such, the exit radius will be 0.08 m, the throat radius will be 0.04 m, and the chamber radius will be 0.02 m.

Subtracting the exit radius by the throat radius over the tangent of 15 degrees yields 0.3021 m for the length of the nozzle from the throat to exit, L_2. Likewise, the length of the nozzle from chamber to throat, L_1, was found in a similar manner, only this time subtracting the radius of the chamber by the radius at the throat. Dividing that value by a tangent of 15 degrees yields a distance of 0.0672 m. These values only deviate slightly from the values gleaned from the NASA software. As such, they are rounded to the nearest first significant digit for the proper dimensions for the finalized design.

$$L_2 = \frac{r_e - r_t}{tan(15^o)} = 0.3021 \text{ m}$$

$$L_1 = \frac{r_c - r_t}{tan(15^o)} = 0.0672 \text{ m}$$

V Results/Observations/Discussions and Figures

V.I Propellant Storage Tank Design

With the calculations made and the engine cycle decided upon (as will be discussed later), it was decided to have two separate storage tanks for the propellant. The fuel tank which holds the liquid methane has an inner radius of 0.32 m and a shell thickness of 0.01 m or 10 mm. Meanwhile, the oxidizer tank for the liquid oxygen has an inner radius of 0.35 m and a shell thickness of 0.01 m.

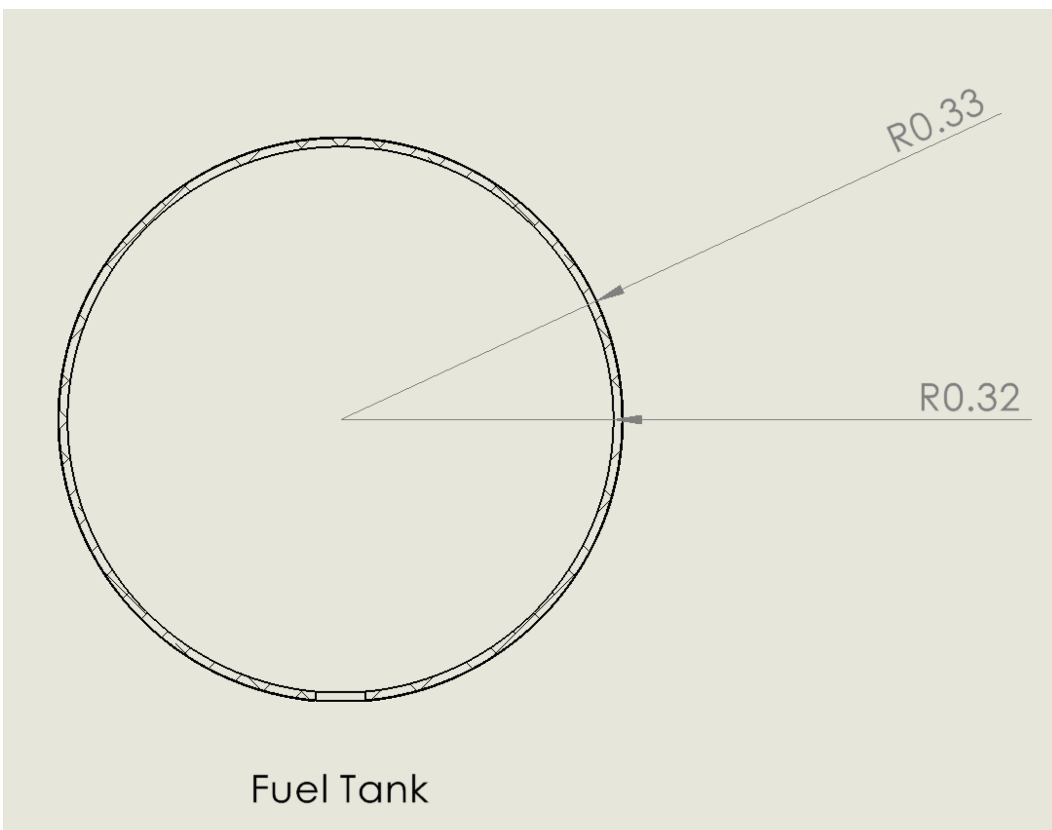

Fig. 5 Fuel Drawing Tank for CH4

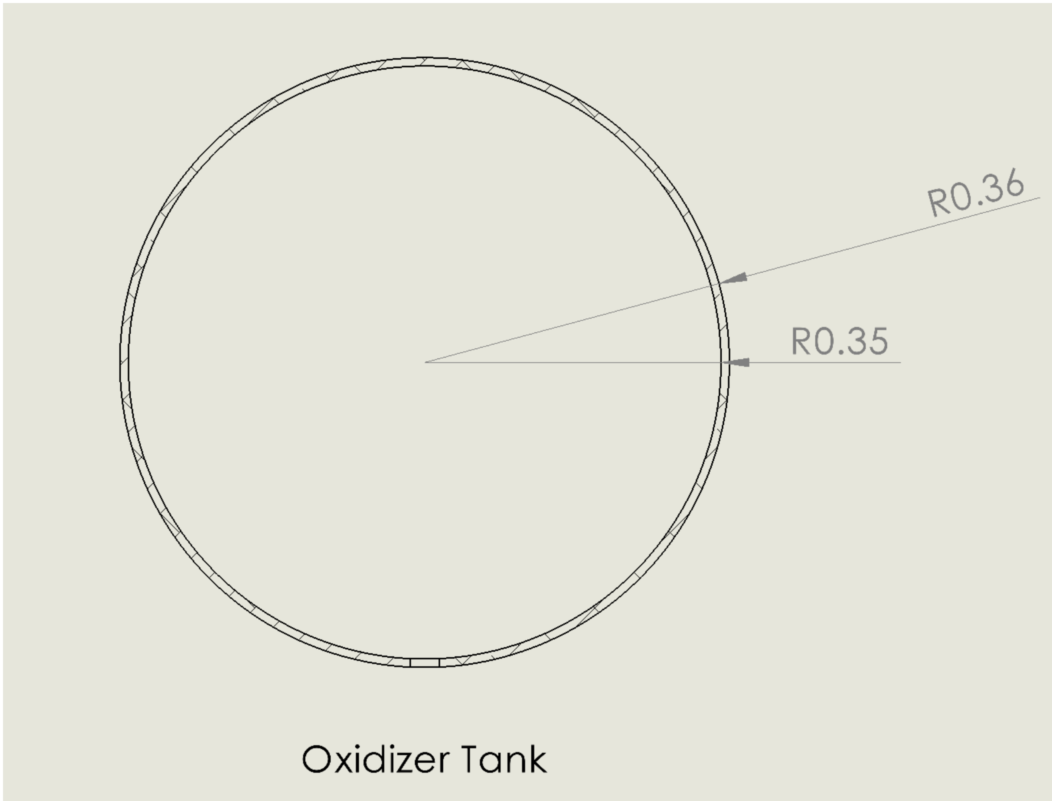

Fig. 6 Oxidizer Drawing Tank for CH4

It was decided that the best material for the fuel tanks was aluminum due to its lightweight nature and high strength. The wall thickness was set to 10mm to ensure stability without using more material then needed. To confirm that this would hold up under the expected tank pressure, simulations were run in ANSYS Mechanical to test it under this force. This was done by adding a fixed support at the exit port to prevent free movement or rotation, and a pressure of 5 bar was applied to the inside face of the tank. The simulation was then run for deformation and equivalent stress. The procedure and results are shown below.

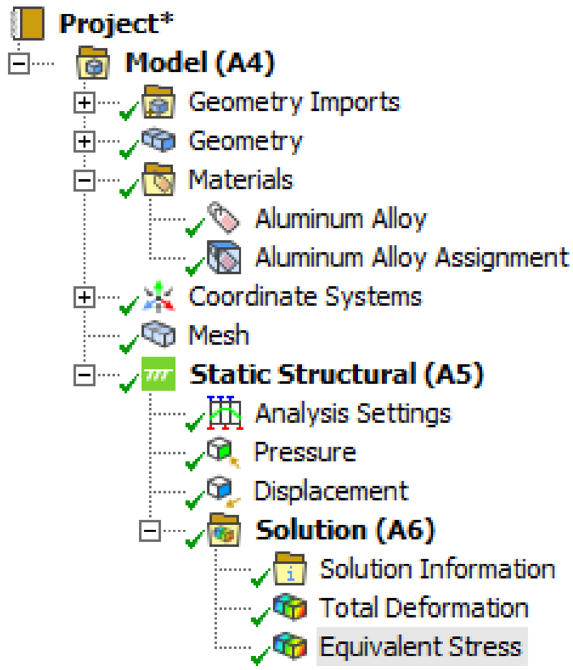

Fig. 7 Simulation Setup Flow Chart for FEA

Scope	
Scoping Method	Geometry Selection
Geometry	2 Faces
Definition	
Type	Pressure
Define By	Normal To
Applied By	Surface Effect
Loaded Area	Deformed
Magnitude	5.e+005 Pa (ramped)
Suppressed	No

Fig. 8 Pressure Setup for FEA

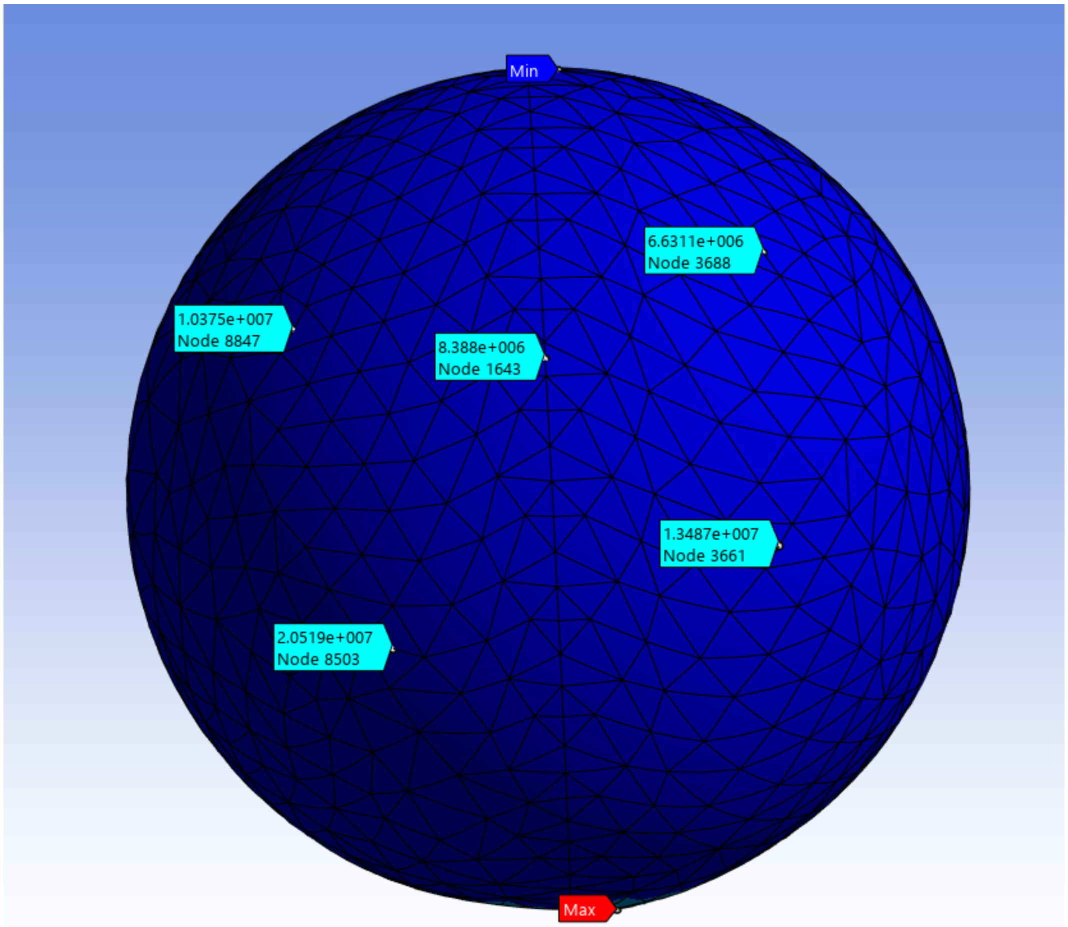

Fig. 9 Results for FEA

These results show that except for the location of the artificial fixed support which produces higher stress then would be seen in reality, all stresses are well below the yield strength of aluminum ensuring that failure is unlikely to occur. There is also a substantial factor of safety. Similar results were shown in the oxidizer tank.

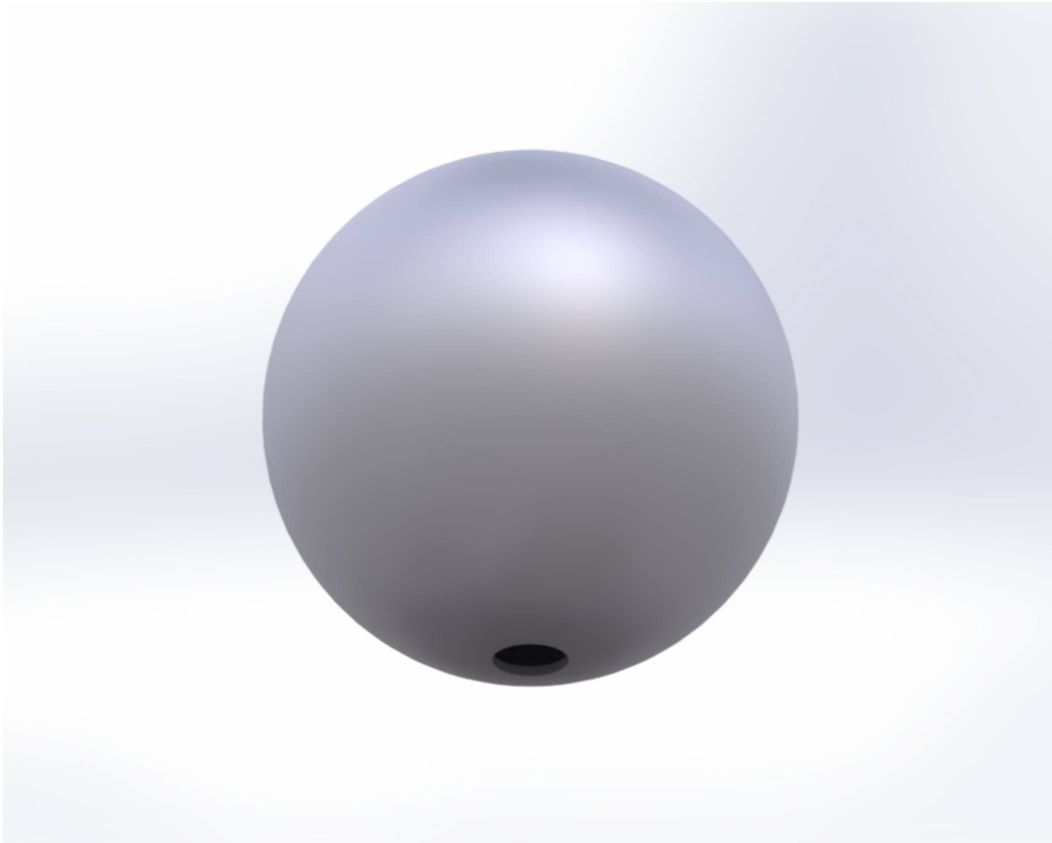

Fig. 10 Fuel Tank Design

V.II Internal Tank Structures

Internal structures of the fuel tanks include screen channels to prevent gases from entering the pump, ring baffles which will prevent slosh in the tank, a bladder to separate fuel/oxidizer from injected gases, and a gas inlet through which pressurized gas will be pumped to force propellant out of the tanks. Piping will also be connected to the tanks running to the pump to transport the propellant. This combination was chosen to ensure that all propellant is able to be utilized with minimal inefficiency or disturbance to the rockets stability and performance.

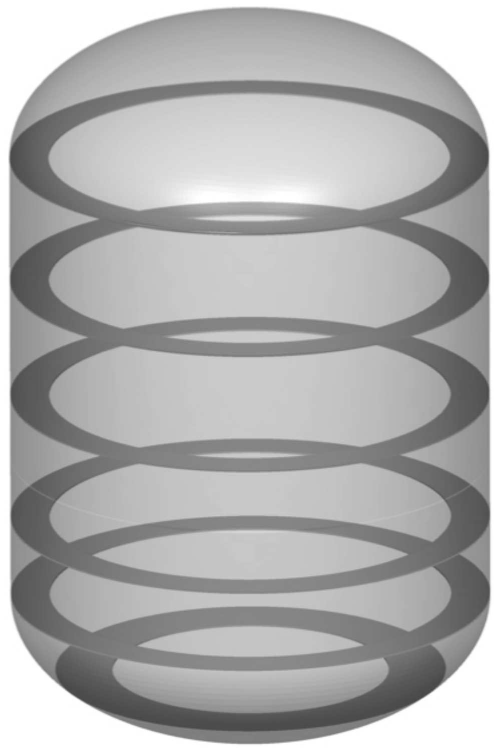

Fig. 11 Ring Baffle Example [16]

V.III Nozzle Design

This information could then be used to design the nozzle. An 85% bell was selected, providing a strong balance of weight and size to fluid efficiency. This was done by calculating the length of each section on the nozzle, converging and diverging, as the difference between each end of the nozzle and the throat, divided by the tangent of 15 degrees. A reasonable curve was then produced between the throat radius and chamber and exit, serving as the inside face of our nozzle. Tangent curves were then produced outside of these at a distance of 10mm to serve as the outside face of the nozzle. This shape was then rotated about the central axis forming the nozzle geometry. Graphite was chosen as the material for the nozzle due to its high strength and thermal resistance in combination with relatively low weight. For the combustion chamber, Inconel was selected for its strong thermal properties and high strength.

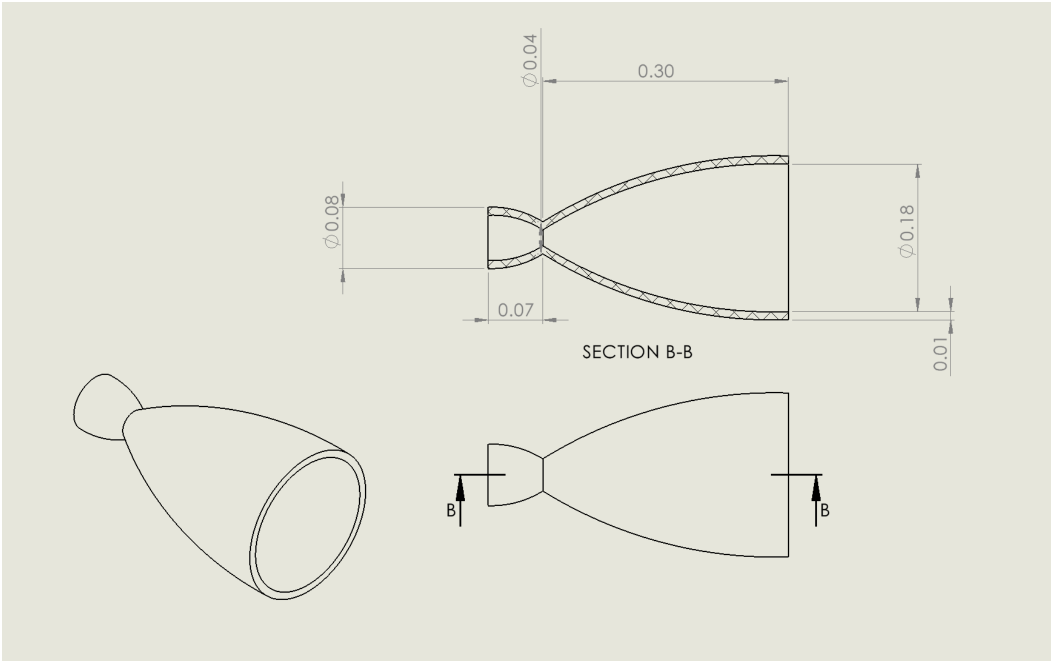

Fig. 12   Nozzle Design

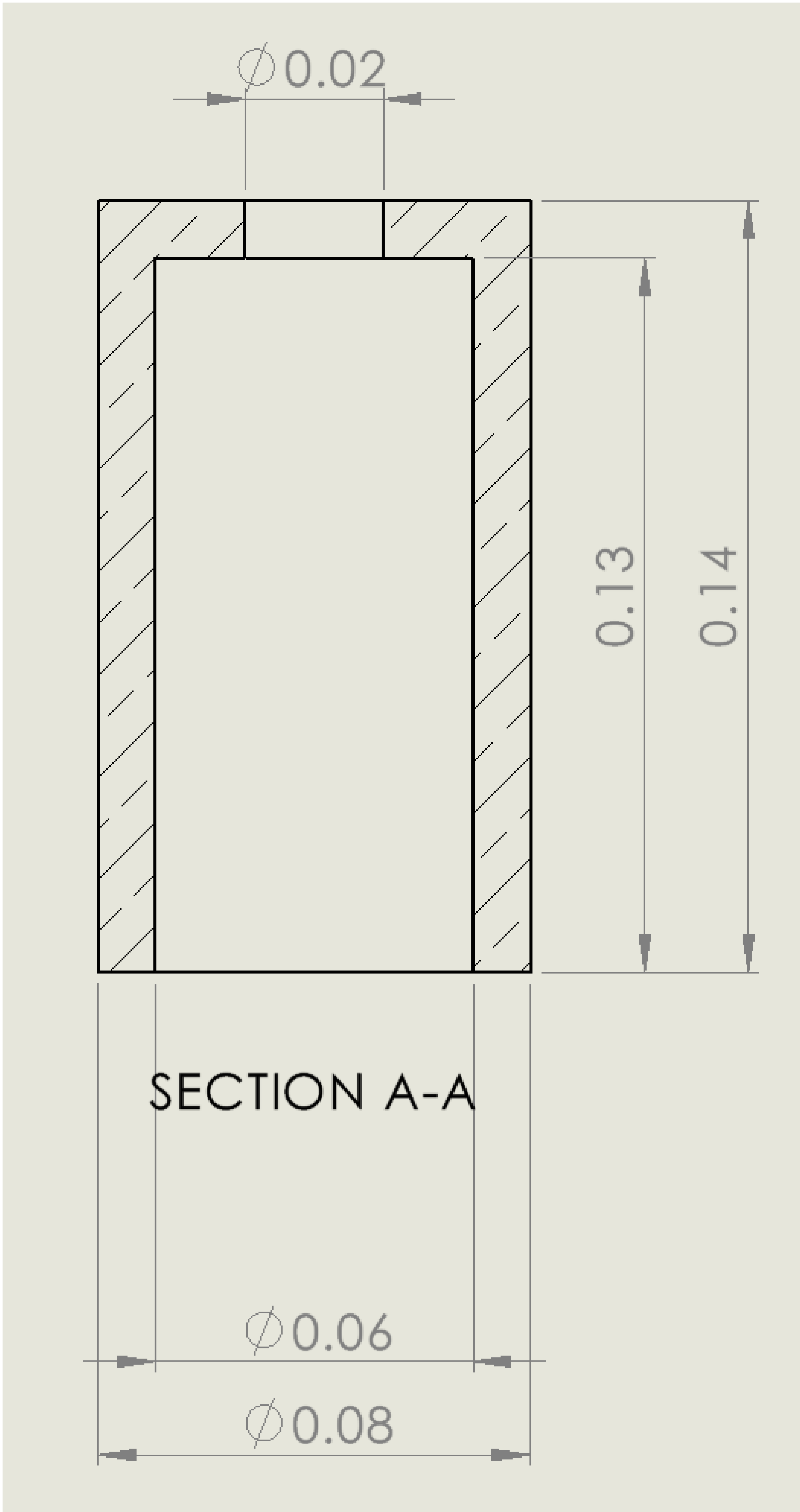

Fig. 13 Chamber Design

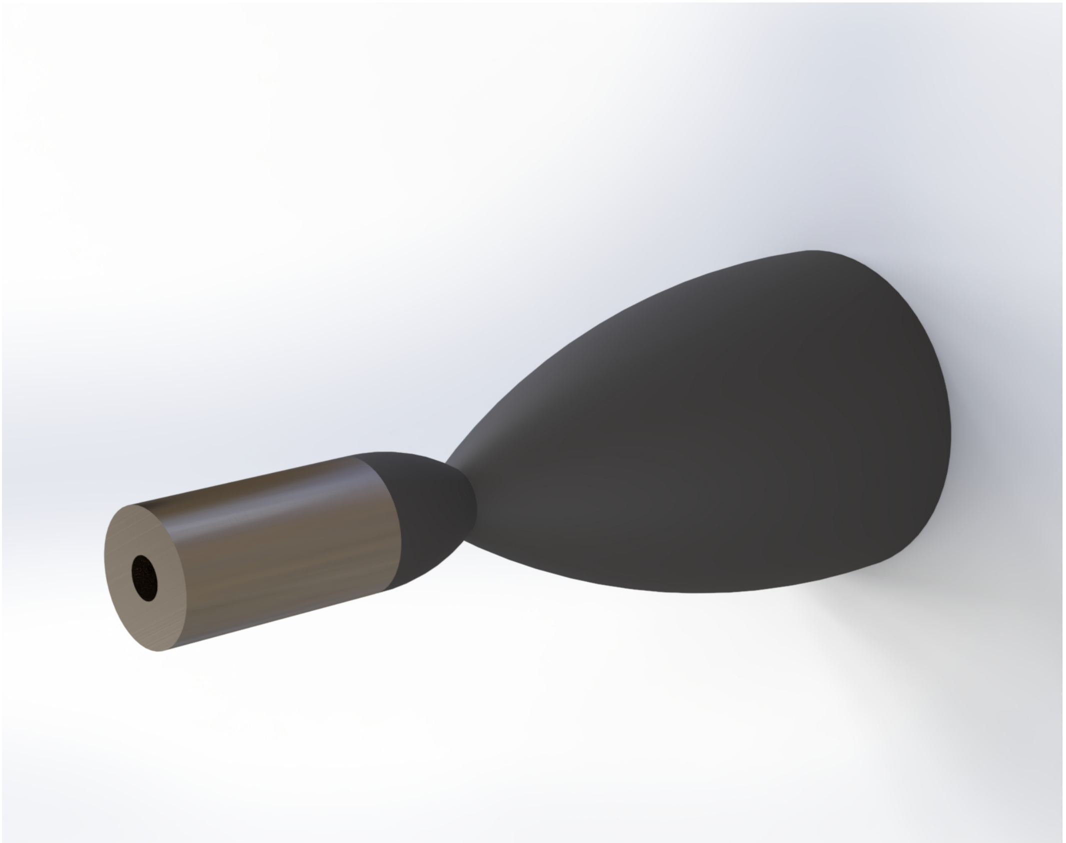

Fig. 14 Nozzle and Chamber Render

VI Engine Cycle

For the rocket, an oxidizer-rich closed-stage combustion engine cycle will be used. In this type of closed-stage combustion, a small amount of oxidizer-rich propellant is burned in the preburner. The preburner, as a result, produces an oxygen-rich hot gas that drives the turbines. With the help of the turbines, the fuel travels to the main combustion chamber. In addition to fuel, the turbine exhaust flow of the oxygen-rich hot gas also travels to the main combustion chamber for further combustion. This ensures that no amount of propellant is wasted and makes the engine produce a higher thrust. Apart from this, the technology behind the oxidizer-rich closed-stage combustion has a lower complexity and a lower turbomachinery count than modern rocket engines. Lastly, this technology is proven and has been perfected by Russia in the 1980s, making it a strong and reliable choice. The BE-4 modern rocket engine uses a similar engine cycle and has an o/f ratio ranging from 3.5:1 to 3.8:1.

This utilization of turbine exhaust gases is what separates the oxidizer-rich closed-stage combustion from the open-stage gas generator cycle. As a result, this engine cycle is more efficient than the gas generator cycle and has a higher chamber pressure. The higher chamber pressure also leads to the engine cycle having a high Isp. The high Isp is critical for this mission to the geostationary orbit because the transfer requires a total delta-v of about 3.9-4 km/s. This is a high amount of delta-v, and it is necessary to ensure no amount of propellant is wasted. Performing a mission to the geostationary orbit also requires a maximization of the payload. This means that using a more efficient rocket engine allows for minimizing propellant weight and maximizing payload weight. The methane-based propellant mixture also ensures a more stable combustion with minimal soot.

As a result of its given advantages, the oxidizer-rich closed-stage engine cycle also has some trade-offs. First and foremost, the hot oxygen-rich gas exiting the preburner is extremely aggressive and reactive. This hot gas then directly sees the turbine, causing damage to the turbine because of its corrosive nature. To mitigate this damage, the materials need to be extremely strong and resistant to the corrosive nature of the gas. Hence, metals such as Nickel and its superalloys, along with multiple protective coatings, are utilised when manufacturing the turbine. Apart from its corrosive nature, the aggressiveness of the oxygen-rich hot gas also requires precise temperature control of the turbine to prevent any damage due to extreme thermal conditions. This precise control of the temperature can be achieved by changing the propellant mixture ratio until a desired temperature is achieved.

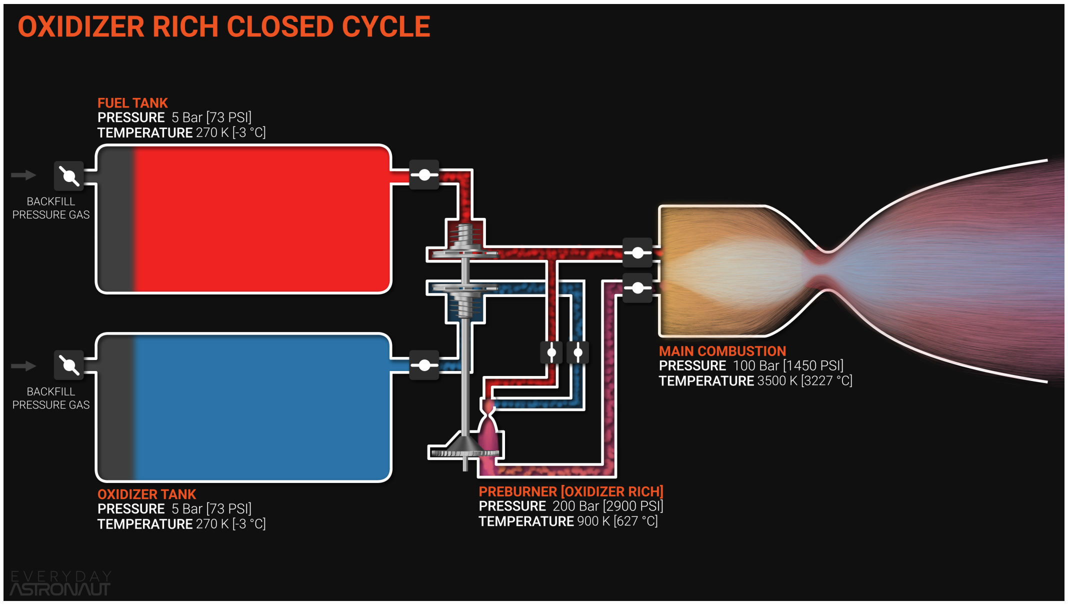

Fig. 15 Oxidizer-rich close-stage combustion cycle diagram

VI.I Feed Mechanism

As stated above, the oxygen-rich closed stage engine cycle will have a preburner where a small amount of oxidizer-rich propellant is burned. After the propellant is burnt, oxygen-rich hot gas exits the preburner, flowing towards the turbine. The gas then spins the turbine and exits through another channel to the main combustion chamber. The spinning turbine is connected to both the LOX pump and the Methane pump via a shaft assembly. The spinning turbine powers the pumps for the propellant, moving the propellant to the main combustion chamber. This type of feed mechanism is proven in use by the Russian engines RD-180 and RD-191.

VI.II Tank Pressurization

The propellant storage tanks are pressurized using Helium. Both the fuel and the oxidizer will have a separate Helium tank at ambient temperatures containing Helium at a pressure of approximately 4500 psi. This value was chosen because the space shuttle program used similar COPV tanks for Helium and Nitrogen, being able to withstand upwards of 5000 psi of pressure. Considering 5000 psi as a maximum limit, a value of 4500 psi was approximated. These COPV tanks in the design will be made out of Kevlar, the same as the space shuttle program. The mechanism will work by the COPV tanks having a valve to regulate flow to the propellant tanks. The propellant tank pressures will be maintained at 5 bar or 500kPa. Hence, as the propellant is used up, Helium will slowly flow into the propellant tanks, utilizing the empty space and pushing the propellant out. A bladder will separate the propellants and the Helium while it pressurizes the tanks.

To calculate the volume of the Helium required for both the fuel and oxidizer, a MATLAB script was used. The following variables were calculated earlier for propellant mass and volume required.

$$\begin{aligned} V_{\text{fuel}} &= 0.1324\,\text{m}^3 \\ V_{\text{oxidizer}} &= 0.1838\,\text{m}^3 \\ P_{\text{tanks}} &= 5\,\text{bar or } 500000\,\text{Pa} \\ P_{\text{He}} &= 4500\,\text{psi or } 31026407.819\,\text{Pa} \\ T_{He} &= 298.15\,\text{K} \\ R_{He} &= 2077\,\frac{J}{kgK} \end{aligned} \tag{1}$$

Additionally, the fuel and oxidizer are assumed to be at their respective boiling points at 5 bars of pressure. This is assumed because liquid Oxygen and Methane have extremely low boiling points, meaning they will be vaporizing almost constantly throughout the mission. Hence, to find the boiling points of Oxygen and Methane at 5 bars of pressure, an online Pearson calculator and PubChem databases were utilized, and the results were as follows:

$$\begin{aligned} T_{\text{fuel}} &= 131.89\,\text{K} \\ T_{\text{oxidizer}} &= 105.98\,\text{K} \end{aligned} \tag{2}$$

The values given above were then used to calculate the mass of Helium required for each tank using the following equation:

$$\begin{aligned} m &= \frac{P_{\text{tanks}} V_{\text{fuel}}}{R_{\text{He}} T_{\text{fuel}}} \\ m_{\text{fuelHe}} &= 0.3625\,\text{kg} \\ m_{\text{oxidizerHe}} &= 0.6262\,\text{kg} \end{aligned} \tag{3}$$

As mentioned earlier, the Helium COPV tank pressures are assumed at 4500 psi, and the temperature is assumed to be 298.15K. That data, along with the mass of Helium, was utilised to calculate the volume of Helium required. The following formula was used:

$$\begin{aligned} V &= \frac{P_{\text{He}} R_{\text{He}} T_{\text{He}}}{P_{He}} \\ V_{\text{fuelHe}} &= 4.823\,\text{liters} \\ V_{\text{oxidizerHe}} &= 8.333\,\text{liters} \end{aligned} \tag{4}$$

However, using Helium to pressurize propellant tanks comes with a risk of a major leakage or the tank bursting. This can result in catastrophic failure of the whole rocket. Moreover, a small leakage would also mean that the volume of Helium present in the system would be lower than the required volume. To address the issue of leakages and bursting the tanks, as mentioned earlier, would be made out of Kevlar, which can withstand pressures up to 5000 psi. Furthermore, to make up for the loss of Helium through minor leakages, a higher volume of Helium will be used than required. Hence, the final volume of Helium with a margin of 1.5 is:

$$\begin{aligned} V_{\text{fuelHe}} &= 7.235\,\text{liters} \\ V_{\text{oxidizerHe}} &= 12.499\,\text{liters} \end{aligned} \tag{5}$$

VII Ignition Mechanism

A torch igniter mechanism will be used to ignite both the main combustion chamber and the preburner for the rocket engine. A torch igniter is like a miniature bi-propellant rocket engine without a combustion chamber and a propellant injector. The torch igniter would use a small flow of methane and oxygen to produce a stable pilot frame. This flame is situated near the main combustion chamber and preburner injection point to ignite the propellant mixture. This is a preferred ignition mechanism for liquid methane rocket engines because methane is a cryogenic fuel that is harder to ignite than the hot gaseous vapors. Hence, a steady torch reliably lights up the combustion chamber despite the cold temperature of the propellant.

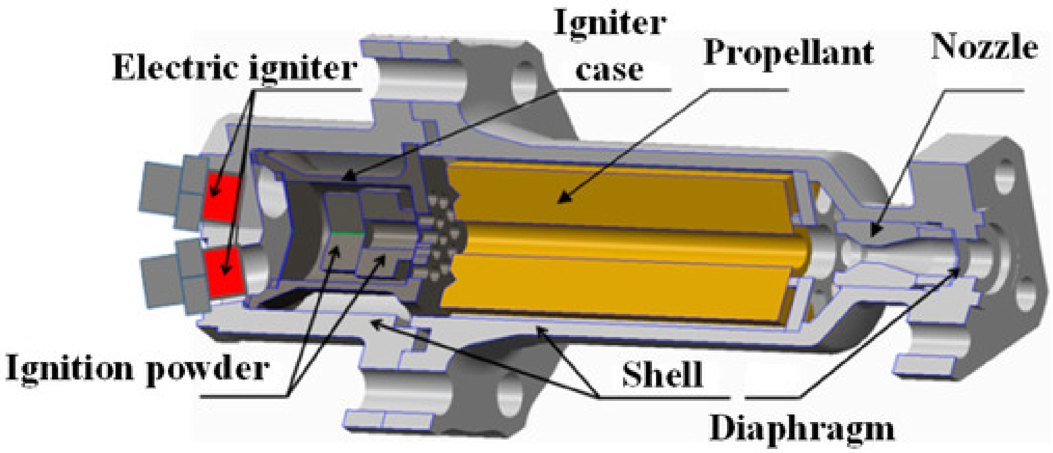

Fig. 16 Typical structure of a torch igniter

In addition to a torch igniter, high energy spark plug will be integrated within the torch igniter. It can not only be used to ignite the torch igniter but also act as a redundancy mechanism in case of failure of the torch igniter. A torch igniter is preferred as the primary ignition mechanism over a high energy spark plug because the cold propellant spray can quench the spark.

VIII Cooling Mechanism

The rocket engine will primarily rely upon regenerative cooling which is a very common method used to cool down the combustion chamber, throat, and the nozzle. The liquid methane will flow through channels and will circulate around the combustion chamber walls, throat, and nozzle before entering the fuel injectors. The channels are built into the walls of the nozzle and the combustion chamber aiding in cooling the engine down. The channels are lined with conductive alloys such as copper-alloys. Being cryogenic, this is an excellent way to cool the engine down as well as heat the fuel up before it enters the combustion chamber. This system is critical because a failure of the system can lead to the nozzle and the combustion chamber melting due to high temperatures. RL-10 engine is an example that uses brazed tube construction for regenerative cooling.

Fig. 17 Example of a 3D printed nozzle with optimized cooling channels

A major disadvantage of this method of cooling is that the pressure inside the walls of the nozzle and chamber has to be higher than the pressure inside the combustion chamber because the fuel has to go from the tubes inside the walls to the fuel injectors. Since pressure always flows from high to low, the pressure inside the walls has to be higher. The increased pressure inside the wall could result in leakages. With fuel flowing directly inside the chamber walls from the tube resulting in overheating of the walls. This is because the propellant mixture for the rocket is 3.6:1 (o/f). To prevent this from happening and further aid in keeping the engine cool, the walls of the nozzle and combustion chamber can be lined with materials that utilize ablative cooling. These materials are made out of carbon composites with extremely high melting points. As the rocket engine burns the propellant, the carbon composite layers are slowly burnt off and vaporized. This is a self-regulating cooling method that is simple and does not require complex moving components. The only disadvantage to the method is that the rocket engines that utilize this type of cooling cannot be reused.

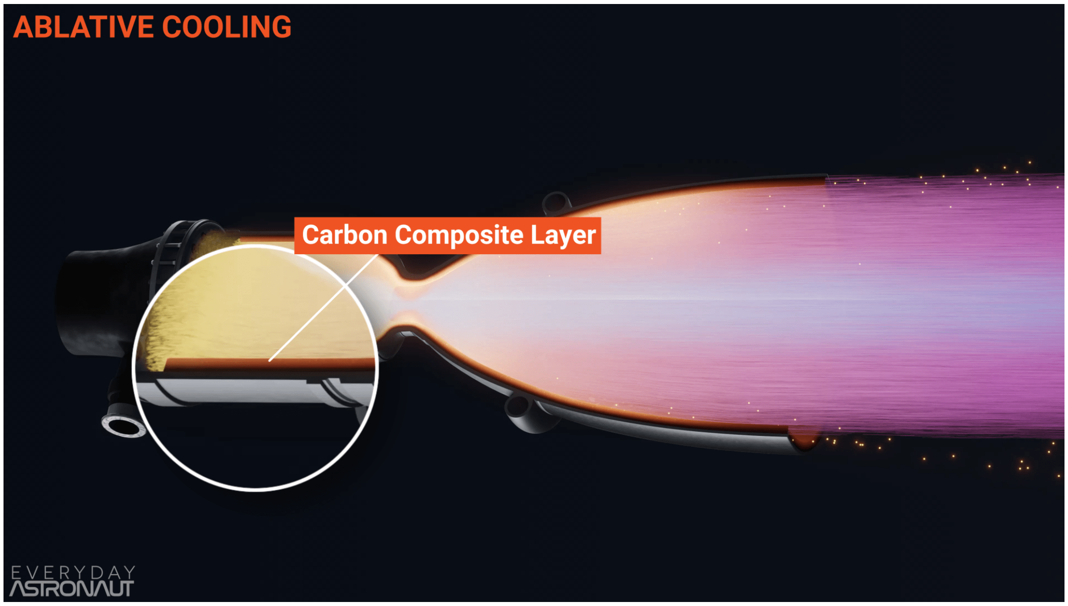

Fig. 18 Diagram showing ablative cooling

Hence, the rocket design would utilize both ablative cooling and regenerative cooling to keep the rocket engine from melting down.

IX Limitation and Uncertainties

Limitations

- **Chamber Temperature Calculations:** The chamber temperature and correlated thermodynamic properties had to be calculated using NASA CEA rather than hand calculations due to the extremely high temperatures involved. Traditional calculation methods produce unrealistic chamber temperatures (e.g., exceeding 5400 K) because of dissociation effects.
- **Assumption of Instantaneous ΔV Burns:** For the purposes of analysis, all ΔV burns were assumed to occur instantaneously. In reality, these maneuvers take place over several seconds, meaning the assumption introduces a small but non-negligible approximation.
- **Lack of Experimental Data:** There is no experimental data for the exact engine and vehicle configuration used in this analysis. Parameters such as critical length therefore had to be estimated from prior or analogous designs.
- **High Mass Ratio:** Due to weight required for structure, the mass ratio had to be higher then would generally be considered ideal for efficiency at 0.33 instead of a more typical 0.2 to 0.1. The mission is still managable under these conditions however.

Uncertainties

- **Fluid Behavior:** Uncertainty remains regarding the detailed fluid behavior within the system, particularly in the nozzle. Thermal or flow inefficiencies may occur and could reduce actual performance relative to predictions.
- **Perturbations:** A range of perturbations may influence the vehicle's flight trajectory, including lunar gravitational interference, radiation effects, and solar positioning. While these effects are expected to be minor, they cannot be fully accounted for in preliminary calculations.
- **Peripheral Structures Integrity:** Structural analysis has been performed for all major components. However, peripheral elements such as piping, vents, or small structural members have not been analyzed to the same degree, introducing some uncertainty in overall system robustness.

X Test Design

The most effective way to test a engine at a small scale such as this would be to utilize a vacuum chamber am perform a static fire test. This will allow specific impulse to be measured from the thrust produced, mass flow rate to be calculated from burn time, temperature would be able to be determined through the use of sensors embedded at key locations. Specialized pressure sensors could be used to measure tank and chamber pressures. These would determine how closely real world parameters match their analytical counterparts enabling any necessary revisions. Additionally, this would enable material testing, all components after the test would be inspected for damage and tested to failure. This would reduce the chance of material failure in a finalized design.

In order to do this, a series of pre-test plans and post-processing tasks must be carried out. Initially, the engine would undergo a detailed pre-test sequence that would ensure all systems are operating accordingly. This would include calibration of thrust stands, cold flow testing, and leak checks. Once the system is validated, the vacuum-fired static fire test could then be performed to collect data regarding thrust, chamber pressure, temperature, and mass-flow data. Once the test is complete, the post-processing would ensue. The engine would first be inspected for any thermal or structural degradation. This is important for reusability purposes as any sign of deformation is something to be accounted for. Lastly, numerical analysis would be performed to compare any experimental results with expected values. This includes identifying if the thrust curve matches its predicted values, verifying if the ignition delay is within limits, and noting if the chamber pressure is stable. Overall, these steps would confirm whether the engine performs as designed and highlight any areas where modifications are needed before implementation occurs at a larger scale.

XI Conclusions

In conclusion, our engine is designed to perform a maneuver for a small spacecraft from the bottom of LEO to GEO where it will carry out its intended purpose. Due to its lightweight nature and the relatively small delta V required, a balance had to be struck between power and efficiency in order to reach the intended distance without burning excessive fuel. Ultimately a methalox configuration was decided on that would be capable of producing the specific impulse needed for this mission. Based on this, the surrounding infrastructure was developed to enable its successful operation and prevent damage or failure.

XII Appendix

XII.I MATLAB Script for calculating delta V required and component dimensions

```
clear all
close all
clc

%gravity
mu = 3.986*10^5;
g0 = 9.81;


%Mission Parameters
m0 = 400;
r1 = 6378 + 160;
r2 = 6378 + 35786;

%Delta V calculations
a = (r1+r2)/2;

v1 = sqrt(mu/r1);
v4 = sqrt(mu/r2);
v2 = sqrt(mu*((2/r1)-(1/a)));
v3 = sqrt(mu*((2/r2)-(1/a)));

delv1 = v2-v1;
delv2 = v4-v3;

delvt = abs(delv1)+abs(delv2);

%Mass Ratios and Propellant Mass
MR = 0.33;
MR2 = 1/MR;
mp = (1-MR)*m0;
rat_mix = 3.6;

%Specific impulse required
I_s = (delvt*1000)/(g0*log(MR2));

%Atomic Masses
am_C = 12.011;
am_H = 1.00784;
am_O = 15.999;

%Calculation of atomic masses for fuel and oxidizer
am_Fuel = am_C+4*am_H;
am_Oxidizer = 4*(am_O*2);

%Calculations for masses of fuel and oxidizer
m_Fuel = mp/4.6;
m_Oxidizer = 3.6*m_Fuel;

%Density of fuel and oxidizer
rho_Fuel = 440;
rho_Oxidizer = 1141;

%Volume calculations for fuel tanks
volume_Fuel = m_Fuel/rho_Fuel;
```

```
volume_Oxidizer = m_Oxidizer/rho_Oxidizer;

%Radius calculations for fuel tanks
r_Fuel = (volume_Fuel/((4/3)*pi))^(1/3);
r_Oxidizer = (volume_Oxidizer/((4/3)*pi))^(1/3);


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Assumptions
rat_contraction = 8;
p_c = 9*10^6;
L_star = 1;
m_dot = 1.5;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Placeholder Stoich Values (REPLACE WHEN FOUND)
T_c = 3598.04;
gamma = 1.13;
R_s = 373.4655;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

v_e = I_s*g0; %Exit velocity
P_ratio = (1-((v_e^2*(gamma-1))/(2*gamma*R_s*T_c)))^(1/((gamma-1)/gamma)); %Pressure
    ratio (p_e/p_c)
T_e = T_c/((1/P_ratio)^((gamma-1)/gamma)); %Exit temperature
a = sqrt(gamma*R_s*T_e); %Speed of sound exit
M_e = v_e/a; %Mach number at exit
rat_expansion = (1/M_e)*sqrt((((1+0.5*(gamma-1)*M_e^2)/(1+0.5*(gamma-1)))^((gamma+1)/(
    gamma-1))));%Expansion ratio
p_e = P_ratio*p_c; %Exit pressure
A_e = (m_dot/v_e)*((R_s*T_e)/p_e); %Exit area
A_t = A_e/rat_expansion; %Throat area
A_c = A_t*rat_contraction; %Chamber area
V_c = A_t*L_star; %Chamber volume
L_c = V_c/A_c; %Length of chamber
r_e = sqrt(A_e/pi); %Exit radius
r_t = sqrt(A_t/pi); %Throat radius
r_c = sqrt(A_c/pi); %Chamber radius
L_2 = (r_e-r_t)/tand(15); %Length of nozzle from throat to exit (85% bell)
L_1 = (r_c-r_t)/tand(15); %Length of nozzle from chamber to throat
```

XII.II MATLAB Script for calculating Helium requirements

```
clear; clc; close all

Vf = 0.1324; % m^3
Vo = 0.1838; % m^3
Pprop = 500000; % Pa
R_He = 2077; % J/kg.K

% Boiling points of oxygen and methane at 5 bar pressure
% Found using pearson calculator
Tf = 131.89; % K
To = 105.98; % K

% Calculating mass of helium
mf_He = (Pprop*Vf)/(R_He*Tf);
mo_He = (Pprop*Vo)/(R_He*To);

% Space shuttles utilised COPVs made of kevlar able to withstand pressures
% of 5000 psi of Helium and Nitrogen. So using 4500 psi as the Helium
% pressure. Using ambient temperature of 298.15K

T_He = 298.15; % K
P_He = 31026407.819; % Pa

% Calculating Helium tank volumes
Vf_He = (mf_He*R_He*T_He)/P_He; % m^3
Vo_He = (mo_He*R_He*T_He)/P_He; % m^3

fprintf('Helium Tank volume for fuel = %.3f litres\n',Vf_He*1000)
fprintf('Helium Tank volume for oxidizer = %.3f litres\n',Vo_He*1000)

% Finding helium tank volumes with a practical margin factor of 1.5
mf_He = 1.5 * mf_He;
mo_He = 1.5 * mo_He;

% Calculating Helium tank volumes with margin factor
Vf_He = (mf_He*R_He*T_He)/P_He; % m^3
Vo_He = (mo_He*R_He*T_He)/P_He; % m^3

fprintf('--------------------------------------------\n')
fprintf('Helium Tank volume for fuel with margin = %.3f litres\n',Vf_He*1000)
fprintf('Helium Tank volume for oxidizer with margin = %.3f litres\n',Vo_He*1000)
```

XIII Hand Calculations for Chamber Temperature

$$CH_4 + 1.8\,O_2 \longrightarrow 0.6\,CO_2 + 0.4\,CO + 1.6\,H_2O + 0.4\,H_2$$

Given:

$$T = 298\,\mathrm{K}$$

$$\Delta H_f^\circ \ (\mathrm{kJ/mol})$$

ΔH_f° **(kJ/mol)**	**Component**
-83	CH_4
0	O_2
-393.522	CO_2
-110.527	CO
-241.826	H_2O
0	H_2

$$\Delta H_r = 0.6(-393.522) + 0.4(-110.527) + 1.6(-241.826) + 0.4(0) - (-83)$$

$$\Delta H_r \approx -750.2456\ \mathrm{kJ/mol}$$

Page 2: CEA Results at 3600 K

$$H - H^\circ(T_r)\ (\mathrm{kJ/mol})$$

$H - H^\circ(T_r)$ **(kJ/mol)**	**Component**
190.393	CO_2
115.978	CO
160.485	H_2O
111.38	H_2

$$H - H^\circ(T_r) = 0.6(190.393) + 0.4(115.978) + 1.6(160.485) + 0.4(111.38)$$

$$H - H^\circ(T_r) \approx 461.955\ \mathrm{kJ/mol}$$

Page 3: 6000 K

$$H - H^\circ(T_r)\ (\mathrm{kJ/mol})$$

343.779	CO_2
207.163	CO
302.295	H_2O
208.341	H_2

$$H - H^\circ(T_r) = 0.6(343.779) + 0.4(207.163) + 1.6(302.295) + 0.4(208.341)$$

$$H - H^\circ(T_r) \approx 856.141\ \mathrm{kJ/mol}$$

Page 4: 5500 K

$H - H^\circ(T_r)$ (kJ/mol)

311.416	CO_2
188.008	CO
272.157	H_2O
187.465	H_2

$$H - H^\circ(T_r) = -772.49$$

Page 5: 5400 K

$H - H^\circ(T_r)$ (kJ/mol)

304.971	CO_2
184.187	CO
266.2	H_2O
183.3	H_2

$$H - H^\circ(T_r) = -755.8974 \text{ kJ/mol} \approx -750.2456 \text{ kJ/mol}$$

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