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Mach 0.01 Half Cylinder

```
clear; clc; close all

% Reading the mesh file
mesh = readmatrix("half_cylinder.txt");

% Reading mesh dimensions
npx = mesh(1,1); % Dimension of mesh
npy = mesh(1,2); % Dimension of mesh
mesh(1,:) = []; % deleting first row

% Sorting mesh coordinates
ndimn = 2;
coord = zeros(ndimn,npx,npy); % pre-allocating coordinate matrix

num = 1; % counts the iteration
for j = 1:npy
    for i = 1:npx
        for k = 1:ndimn
            coord(k,i,j) = mesh(num,k);
        end
        num = num + 1; % updating variable
    end
end

% Subtracting one from number of nodes given in rows in and columns to
% obtain the number of cells in row and column
ncx = npx - 1; % number of cells in row
ncy = npy - 1; % number of cells in column

%Sanity grid plot check
x_val = squeeze(coord(1, :, :));
y_val = squeeze(coord(2, :, :));

figure(1)
plot(x_val,y_val, 'k-');
hold on
plot(x_val',y_val', 'k-');
xlabel('x')
ylabel('y')
title('Half Cylinder Plot Check')
axis equal tight
```

```

% Geometry Calculations Using geo_calc.m function
geoel = geo_calc(coord);

% Setting up initial conditions
% Using standard atmospheric conditions as instructed. Gamma for air is
% about 1.4 and R for air is 287 J/Kg*K. Temperature and pressure at
% freestream conditions are given along with a Mach range.

T_inf = 300; % K
P_inf = 101325; % Pa
gamma = 1.4; % For air
R = 287; % J/Kg*K
Mach = 0.01;

% Freestream density
rho_inf = P_inf/(R*T_inf); % ideal gas law kg/m^3

% Iteration settings
omega = 1;
iter = 10000;
tol = 1e-4; % convergence criterion

% Speed of sound calculations
a_inf = sqrt(gamma*R*T_inf); % m/s

% Finding initial speed using freestream speed of sound and Mach numbers. u_inf
% is non-zero because I assuming the fluid freestream moves from left to
% right along the x-axis. v_inf is zero because I am assuming zero
% freestream velocity upwards or in the y-axis.
u_inf = a_inf*Mach; % m/s
v_inf = 0;

% Creating array for unknowns such as phi and rho and velocity. Adding
% 2 to the number of cells for ghost cell layer around all the walls.
var = 4; % 1 phi, 2 u-comp, 3 v-comp, 4 density
unkno = zeros(var, ncx+2, ncy+2);

% Initial guess for velocity potential based on freestream velocity
% components. The velocity potential for each cell will be at the cell
% centroid. Would help this way because the array is cell centered in
% scrap code.
for k = 1:ncy
    for j = 1:ncx
        x_cent = geoel(1,j+1,k+1); % centroid for cells
        y_cent = geoel(2,j+1,k+1); % centroid for cells

        % Same initial guess for all real cells.
        unkno(1,j+1,k+1) = u_inf*x_cent + v_inf*y_cent;
    end
end

% Initializing density values
unkno(4,2:ncx+1,2:ncy+1) = rho_inf;

```

```

% Initializing residual history
resi_hist = zeros(iter,1);

% Iteration loop for convergence
for num = 1:iter

    % Ghost cell boundary conditions application
    unkno = boundary_cond_cylinder(unkno,geoel,u_inf,v_inf);

    % Calculating velocity and density values from phi values for both real
    % and ghost cells
    unkno = DensityfromPhi(unkno,geoel,gamma,Mach,a_inf,rho_inf);

    % Calculating the residuals and the coefficients
    [resi,coeff] = resi_calc(unkno,geoel);

    % Updating phi using Gauss-Seidel Method
    [unkno,dphi,resi_n] = updatePhi(unkno,resi,coeff,omega);

    % Storing residual history
    resi_hist(num) = resi_n;

    % Convergence check
    if num == 1
        resi0 = resi_n;
    end

    resi_temp = resi_n/resi0;

    if resi_temp < tol
        resi_hist = resi_hist(1:num);
        break
    end
end

if num == iter
    resi_hist = resi_hist(1:iter);
end

% Extracting data for plotting
Xc = squeeze(geoel(1,2:ncx+1,2:ncy+1));
Yc = squeeze(geoel(2,2:ncx+1,2:ncy+1));
Uc = squeeze(unkno(2,2:ncx+1,2:ncy+1));
Vc = squeeze(unkno(3,2:ncx+1,2:ncy+1));
rhoc = squeeze(unkno(4,2:ncx+1,2:ncy+1));

% Velocity magnitude
Vmag = sqrt(Uc.^2 + Vc.^2);

% Local mach number
Mach_local = Vmag./a_inf;

Vwall = sqrt(Uc(:,1).^2 + Vc(:,1).^2);
Mwall = Vwall/a_inf;

```

```

% Pressure coefficient calculation
numerator = 1 + (gamma-1)/2 * Mach^2;
denominator = 1 + (gamma-1)/2 * Mwall.^2;
Cpwall = 2/(gamma*Mach^2).*((numerator./denominator).^(gamma/(gamma-1)) - 1);

% X/c along the wall
Xwall = Xc(:,1);
Xwall = Xwall ./ max(abs(Xwall));

% Sort for clean plotting
[Xwall, indW] = sort(Xwall);
Mwall = Mwall(indW);
Cpwall = Cpwall(indW);

% Plots
figure()
plot(1:length(resi_hist),resi_hist,'LineWidth',2)
grid on
xlabel('No. of Iterations')
ylabel('Residual')
title('Half Cylinder Residual, Mach = 0.01, Omega = 1, Iter = 3043')

figure()
contourf(Xc',Yc',Vmag',30,'LineStyle','none')
hold on
colorbar
quiver(Xc',Yc',Uc',Vc','k')
axis equal tight
title('Half Cylinder, Mach = 0.01, Omega = 1, Iter = 3043')

figure()
plot(Xwall,Mwall,'LineWidth',1.5)
xlabel('X/c')
ylabel('Mach Number')
title('Half Cylinder, Mach Number along the Body Wall')
grid on

figure()
plot(Xwall,Cpwall,'LineWidth',1.5)
xlabel('X/c')
ylabel('Pressure Coefficient')
title('Half Cylinder, Pressure coefficient along the Body Wall')
grid on

```

Mach 0.5 Half Cylinder

```

clear; clc; close all

% Reading the mesh file
mesh = readmatrix("half_cylinder.txt");

% Reading mesh dimensions

```

```

npx = mesh(1,1); % Dimension of mesh
npy = mesh(1,2); % Dimension of mesh
mesh(1,:) = []; % deleting first row

% Sorting mesh coordinates
ndimn = 2;
coord = zeros(ndimn,npx,npy); % pre-allocating coordinate matrix

num = 1; % counts the iteration
for j = 1:npy
    for i = 1:npx
        for k = 1:ndimn
            coord(k,i,j) = mesh(num,k);
        end
        num = num + 1; % updating variable
    end
end

% Subtracting one from number of nodes given in rows in and columns to
% obtain the number of cells in row and column
ncx = npx - 1; % number of cells in row
ncy = npy - 1; % number of cells in column

%Sanity grid plot check
x_val = squeeze(coord(1, :, :));
y_val = squeeze(coord(2, :, :));

figure(1)
plot(x_val,y_val, 'k-');
hold on
plot(x_val',y_val', 'k-');
xlabel('x')
ylabel('y')
title('Half Cylinder Plot Check')
axis equal tight

% Geometry Calculations Using geo_calc.m function
geoel = geo_calc(coord);

% Setting up initial conditions
% Using standard atmospheric conditions as instructed. Gamma for air is
% about 1.4 and R for air is 287 J/Kg*K. Temperature and pressure at
% freestream conditions are given along with a Mach range.

T_inf = 300; % K
P_inf = 101325; % Pa
gamma = 1.4; % For air
R = 287; % J/Kg*K
Mach = 0.5;

% Freestream density
rho_inf = P_inf/(R*T_inf); % ideal gas law kg/m^3

% Iteration settings

```

```

omega = 0.03;
iter = 18000;
tol = 1e-4; % convergence criterion

% Speed of sound calculations
a_inf = sqrt(gamma*R*T_inf); % m/s

% Finding initial speed using freestream speed of sound and Mach numbers. u_inf
% is non-zero because I assuming the fluid freestream moves from left to
% right along the x-axis. v_inf is zero because I am assuming zero
% freestream velocity upwards or in the y-axis.
u_inf = a_inf*Mach; % m/s
v_inf = 0;

% Creating array for unknowns such as phi and rho and velocity. Adding
% 2 to the number of cells for ghost cell layer around all the walls.
var = 4; % 1 phi, 2 u-comp, 3 v-comp, 4 density
unkno = zeros(var, ncx+2, ncy+2);

% Initial guess for velocity potential based on freestream velocity
% components. The velocity potential for each cell will be at the cell
% centroid. Would help this way because the array is cell centered in
% scrap code.
for k = 1:ncy
    for j = 1:ncx
        x_cent = geoel(1,j+1,k+1); % centroid for cells
        y_cent = geoel(2,j+1,k+1); % centroid for cells

        % Same initial guess for all real cells.
        unkno(1,j+1,k+1) = u_inf*x_cent + v_inf*y_cent;
    end
end

% Initializing density values
unkno(4,2:ncx+1,2:ncy+1) = rho_inf;

% Initializing residual history
resi_hist = zeros(iter,1);

% Iteration loop for convergence
for num = 1:iter

    % Ghost cell boundary conditions application
    unkno = boundary_cond_cylinder(unkno,geoel,u_inf,v_inf);

    % Calculating velocity and density values from phi values for both real
    % and ghost cells
    unkno = DensityfromPhi(unkno,geoel,gamma,Mach,a_inf,rho_inf);

    % Calculating the residuals and the coefficients
    [resi,coeff] = resi_calc(unkno,geoel);

    % Updating phi using Gauss-Seidel Method
    [unkno,dphi,resi_n] = updatePhi(unkno,resi,coeff,omega);

```

```

% Storing residual history
resi_hist(num) = resi_n;

% Convergence check
if num == 1
    resi0 = resi_n;
end

resi_temp = resi_n/resi0;

if resi_temp < tol
    resi_hist = resi_hist(1:num);
    break
end
end

if num == iter
    resi_hist = resi_hist(1:iter);
end

% Extracting data for plotting
Xc = squeeze(geoel(1,2:ncx+1,2:ncy+1));
Yc = squeeze(geoel(2,2:ncx+1,2:ncy+1));
Uc = squeeze(unkno(2,2:ncx+1,2:ncy+1));
Vc = squeeze(unkno(3,2:ncx+1,2:ncy+1));
rhoc = squeeze(unkno(4,2:ncx+1,2:ncy+1));

% Velocity magnitude
Vmag = sqrt(Uc.^2 + Vc.^2);

% Local mach number
Mach_local = Vmag./a_inf;

Vwall = sqrt(Uc(:,1).^2 + Vc(:,1).^2);
Mwall = Vwall/a_inf;

% Pressure coefficient calculation
numerator = 1 + (gamma-1)/2 * Mach^2;
denominator = 1 + (gamma-1)/2 * Mwall.^2;
Cpwall = 2/(gamma*Mach^2).*((numerator./denominator).^(gamma/(gamma-1)) - 1);

% X/c along the wall
Xwall = Xc(:,1);
Xwall = Xwall ./ max(abs(Xwall));

% Sort for clean plotting
[Xwall, indW] = sort(Xwall);
Mwall = Mwall(indW);
Cpwall = Cpwall(indW);

% Plots
figure()
plot(1:length(resi_hist),resi_hist,'LineWidth',2)

```

```
grid on
xlabel('No. of Iterations')
ylabel('Residual')
title('Half Cylinder Residual, Mach = 0.5, Omega = 0.03, Iter = 18000')

figure()
contourf(Xc',Yc',Vmag',30,'LineStyle','none')
hold on
colorbar
quiver(Xc',Yc',Uc',Vc','k')
axis equal tight
title('Half Cylinder, Mach = 0.5, Omega = 0.03, Iter = 18000')

figure()
plot(Xwall,Mwall,'LineWidth',1.5)
xlabel('X/c')
ylabel('Mach Number')
title('Half Cylinder, Mach Number along the Body Wall')
grid on

figure()
plot(Xwall,Cpwall,'LineWidth',1.5)
xlabel('X/c')
ylabel('Pressure Coefficient')
title('Half Cylinder, Pressure coefficient along the Body Wall')
grid on
```

Mach 0.8 Half Cylinder

```
clear; clc; close all

% Reading the mesh file
mesh = readmatrix("half_cylinder.txt");

% Reading mesh dimensions
npx = mesh(1,1); % Dimension of mesh
npy = mesh(1,2); % Dimension of mesh
mesh(1,:) = []; % deleting first row

% Sorting mesh coordinates
ndimn = 2;
coord = zeros(ndimn,npx,npy); % pre-allocating coordinate matrix

num = 1; % counts the iteration
for j = 1:npy
    for i = 1:npx
        for k = 1:ndimn
            coord(k,i,j) = mesh(num,k);
        end
        num = num + 1; % updating variable
    end
end
```

```

% Subtracting one from number of nodes given in rows in and columns to
% obtain the number of cells in row and column
ncx = npx - 1; % number of cells in row
ncy = npy - 1; % number of cells in column

%Sanity grid plot check
x_val = squeeze(coord(1, :, :));
y_val = squeeze(coord(2, :, :));

figure(1)
plot(x_val, y_val, 'k-');
hold on
plot(x_val', y_val', 'k-');
xlabel('x')
ylabel('y')
title('Half Cylinder Plot Check')
axis equal tight

% Geometry Calculations Using geo_calc.m function
geoel = geo_calc(coord);

% Setting up initial conditions
% Using standard atmospheric conditions as instructed. Gamma for air is
% about 1.4 and R for air is 287 J/Kg*K. Temperature and pressure at
% freestream conditions are given along with a Mach range.

T_inf = 300; % K
P_inf = 101325; % Pa
gamma = 1.4; % For air
R = 287; % J/Kg*K
Mach = 0.8;

% Freestream density
rho_inf = P_inf/(R*T_inf); % ideal gas law kg/m^3

% Iteration settings
omega = 0.008;
iter = 16000;
tol = 1e-4; % convergence criterion

% Speed of sound calculations
a_inf = sqrt(gamma*R*T_inf); % m/s

% Finding initial speed using freestream speed of sound and Mach numbers. u_inf
% is non-zero because I assuming the fluid freestream moves from left to
% right along the x-axis. v_inf is zero because I am assuming zero
% freestream velocity upwards or in the y-axis.
u_inf = a_inf*Mach; % m/s
v_inf = 0;

% Creating array for unknowns such as phi and rho and velocity. Adding
% 2 to the number of cells for ghost cell layer around all the walls.
var = 4; % 1 phi, 2 u-comp, 3 v-comp, 4 density
unkno = zeros(var, ncx+2, ncy+2);

```

```

% Initial guess for velocity potential based on freestream velocity
% components. The velocity potential for each cell will be at the cell
% centroid. Would help this way because the array is cell centered in
% scrap code.
for k = 1:ncy
    for j = 1:ncx
        x_cent = geoel(1,j+1,k+1); % centroid for cells
        y_cent = geoel(2,j+1,k+1); % centroid for cells

        % Same initial guess for all real cells.
        unkno(1,j+1,k+1) = u_inf*x_cent + v_inf*y_cent;
    end
end

% Initializing density values
unkno(4,2:ncx+1,2:ncy+1) = rho_inf;

% Initializing residual history
resi_hist = zeros(iter,1);

% Iteration loop for convergence
for num = 1:iter

    % Ghost cell boundary conditions application
    unkno = boundary_cond_cylinder(unkno,geoel,u_inf,v_inf);

    % Calculating velocity and density values from phi values for both real
    % and ghost cells
    unkno = DensityfromPhi(unkno,geoel,gamma,Mach,a_inf,rho_inf);

    % Calculating the residuals and the coefficients
    [resi,coeff] = resi_calc(unkno,geoel);

    % Updating phi using Gauss-Seidel Method
    [unkno,dphi,resi_n] = updatePhi(unkno,resi,coeff,omega);

    % Storing residual history
    resi_hist(num) = resi_n;

    % Convergence check
    if num == 1
        resi0 = resi_n;
    end

    resi_temp = resi_n/resi0;

    if resi_temp < tol
        resi_hist = resi_hist(1:num);
        break
    end
end

if num == iter

```

```

    resi_hist = resi_hist(1:iter);
end

% Extracting data for plotting
Xc = squeeze(geoel(1,2:ncx+1,2:ncy+1));
Yc = squeeze(geoel(2,2:ncx+1,2:ncy+1));
Uc = squeeze(unkno(2,2:ncx+1,2:ncy+1));
Vc = squeeze(unkno(3,2:ncx+1,2:ncy+1));
rhoc = squeeze(unkno(4,2:ncx+1,2:ncy+1));

% Velocity magnitude
Vmag = sqrt(Uc.^2 + Vc.^2);

% Local mach number
Mach_local = Vmag./a_inf;

Vwall = sqrt(Uc(:,1).^2 + Vc(:,1).^2);
Mwall = Vwall/a_inf;

% Pressure coefficient calculation
numerator = 1 + (gamma-1)/2 * Mach^2;
denominator = 1 + (gamma-1)/2 * Mwall.^2;
Cpwall = 2/(gamma*Mach^2).*((numerator./denominator).^(gamma/(gamma-1)) - 1);

% X/c along the wall
Xwall = Xc(:,1);
Xwall = Xwall ./ max(abs(Xwall));

% Sort for clean plotting
[Xwall, indW] = sort(Xwall);
Mwall = Mwall(indW);
Cpwall = Cpwall(indW);

% Plots
figure()
plot(1:length(resi_hist),resi_hist,'LineWidth',2)
grid on
xlabel('No. of Iterations')
ylabel('Residual')
title('Half Cylinder Residual, Mach = 0.8, Omega = 0.008, Iter = 16000')

figure()
contourf(Xc',Yc',Vmag',30,'LineStyle','none')
hold on
colorbar
quiver(Xc',Yc',Uc',Vc','k')
axis equal tight
title('Half Cylinder, Mach = 0.8, Omega = 0.008, Iter = 16000')

figure()
plot(Xwall,Mwall,'LineWidth',1.5)
xlabel('X/c')
ylabel('Mach Number')
title('Half Cylinder, Mach Number along the Body Wall')

```

```
grid on
```

```
figure()
plot(Xwall,Cpwall, 'LineWidth',1.5)
xlabel('X/c')
ylabel('Pressure Coefficient')
title('Half Cylinder, Pressure coefficient along the Body Wall')
grid on
```

Functions

This function calculates basic geometry of cells within a mesh such as area, centroid, and finding normal vector. It was taken from HW_5 but 3rd and 4th corner definitions were changed so that it is counterclockwise as the number rises. It also will calculate the ghost cell geometry.

```
function geoel = geo_calc(coord)

[ndimn, npx, npy] = size(coord);

ncx = npx - 1; % number of cells in row
ncy = npy - 1; % number of cells in column

ngeel = 11; % No. of geometric calculations
geoel = zeros(ngeel, ncx+2, ncy+2);

for j = 1:ncy
    for i = 1:ncx
        for k = 1:ndimn
            switch k
                case 1
                    % X-coordinates
                    x1 = coord(k,i,j); % Bottom left corner
                    x2 = coord(k,i+1,j); % Bottom right corner
                    x3 = coord(k,i+1,j+1); % Top right corner
                    x4 = coord(k,i,j+1); % Top left corner

                case 2
                    % Y-coordinates
                    y1 = coord(k,i,j); % Bottom left corner
                    y2 = coord(k,i+1,j); % Bottom right corner
                    y3 = coord(k,i+1,j+1); % Top right corner
                    y4 = coord(k,i,j+1); % Top left corner
            end
        end

        % Variables to account for ghost cell layer.
        gi = i + 1;
        gj = j + 1;

        % Cell-centroid box average method
        xc = (x1 + x2 + x3 + x4)/4; % x-coordinate for centroid
        yc = (y1 + y2 + y3 + y4)/4; % y-coordinate for centroid
```

```

% Cell area
area = 0.5*((x3-x1)*(y4-y2)-(x4-x2)*(y3-y1));

% Length weighted normal vector calculation
% North
% Swapping and negating to find the normal vector
nNx = y4 - y3;
nNy = -(x4 - x3);

% East
% Swapping and negating to find the normal vector
nEx = y3 - y2;
nEy = -(x3 - x2);

% South
% Swapping and negating to find the normal vector
nSx = y2 - y1;
nSy = -(x2 - x1);

% West
% Swapping and negating to find the normal vector
nWx = y1 - y4;
nWy = -(x1 - x4);

% Storing values
geoel(1,gi,gj) = xc; % Cell centroid x-axis
geoel(2,gi,gj) = yc; % Cell centroid y-axis
geoel(3,gi,gj) = area; % Cell area
geoel(4,gi,gj) = nEx; % East length weighted normal vector
geoel(5,gi,gj) = nEy; % East length weighted normal vector
geoel(6,gi,gj) = nNx; % North length weighted normal vector
geoel(7,gi,gj) = nNy; % North length weighted normal vector
geoel(8,gi,gj) = nWx; % West length weighted normal vector
geoel(9,gi,gj) = nWy; % West length weighted normal vector
geoel(10,gi,gj) = nSx; % South length weighted normal vector
geoel(11,gi,gj) = nSy; % South length weighted normal vector
end
end

% South Boundary ghost cells

for i = 1:ncx

    x1 = coord(1,i,1);
    x2 = coord(1,i+1,1);
    y1 = coord(2,i,1);
    y2 = coord(2,i+1,1);

    % Centroid for cells above south ghost cell boundary
    xc = geoel(1,i+1,2);
    yc = geoel(2,i+1,2);

    % External area weight normal vector to this face
    rnx = y2 - y1;

```

```

    rny = x1 - x2;

    geoel(6,i+1,1) = -rnx;
    geoel(7,i+1,1) = -rny;

    % Magnitude of the normal vector
    rnmag = sqrt(rnx^2 + rny^2);

    % Unit Normal vector
    rux = rnx/rnmag;
    ruy = rny/rnmag;

    % Normal distance of vector (x1 - xc)
    distn = 2.0*((x1 - xc)*rux + (y1 - yc)*ruy);

    % Ghost cell center coordinates
    geoel(1,i+1,1) = xc + distn*rux;
    geoel(2,i+1,1) = yc + distn*ruy;

    % Ghost cell area
    geoel(3,i+1,1) = geoel(3,i+1,2);
end

% North Boundary ghost cells

for i = 1:ncx

    x1 = coord(1,i,npj);
    x2 = coord(1,i+1,npj);
    y1 = coord(2,i,npj);
    y2 = coord(2,i+1,npj);

    % Centroid for cells below north ghost cell boundary
    xc = geoel(1,i+1,ncy+1);
    yc = geoel(2,i+1,ncy+1);

    % External area weight normal vector to this face
    rnx = y1 - y2;
    rny = x2 - x1;

    geoel(10,i+1,ncy+2) = -rnx;
    geoel(11,i+1,ncy+2) = -rny;

    % Magnitude of the normal vector
    rnmag = sqrt(rnx^2 + rny^2);

    % Unit Normal vector
    rux = rnx/rnmag;
    ruy = rny/rnmag;

    % Normal distance of vector (x1 - xc)
    distn = 2.0*((x1 - xc)*rux + (y1 - yc)*ruy);

    % Ghost cell center coordinates

```

```

    geoel(1,i+1,ncy+2) = xc + distn*rux;
    geoel(2,i+1,ncy+2) = yc + distn*ruy;

    % Ghost cell area
    geoel(3,i+1,ncy+2) = geoel(3,i+1,ncy+1);
end

% West Boundary ghost cells

for j = 1:ncy

    x1 = coord(1,1,j);
    x2 = coord(1,1,j+1);
    y1 = coord(2,1,j);
    y2 = coord(2,1,j+1);

    % Centroid for cells beside west ghost cell boundary
    xc = geoel(1,2,j+1);
    yc = geoel(2,2,j+1);

    % External area weight normal vector to this face
    rnx = y1 - y2;
    rny = x2 - x1;

    geoel(4,1,j+1) = -rnx;
    geoel(5,1,j+1) = -rny;

    % Magnitude of the normal vector
    rnmag = sqrt(rnx^2 + rny^2);

    % Unit Normal vector
    rux = rnx/rnmag;
    ruy = rny/rnmag;

    % Normal distance of vector (x1 - xc)
    distn = 2.0*((x1 - xc)*rux + (y1 - yc)*ruy);

    % Ghost cell center coordinates
    geoel(1,1,j+1) = xc + distn*rux;
    geoel(2,1,j+1) = yc + distn*ruy;

    % Ghost cell area
    geoel(3,1,j+1) = geoel(3,2,j+1);
end

% East Boundary ghost cells

for j = 1:ncy

    x1 = coord(1,npx,j);
    x2 = coord(1,npx,j+1);
    y1 = coord(2,npx,j);
    y2 = coord(2,npx,j+1);

```

```

    % Centroid for cells beside east ghost cell boundary
    xc = geoel(1,ncx+1,j+1);
    yc = geoel(2,ncx+1,j+1);

    % External area weight normal vector to this face
    rnx = y2 - y1;
    rny = x1 - x2;

    geoel(8,ncx+2,j+1) = -rnx;
    geoel(9,ncx+2,j+1) = -rny;

    % Magnitude of the normal vector
    rnmag = sqrt(rnx^2 + rny^2);

    % Unit Normal vector
    rux = rnx/rnmag;
    ruy = rny/rnmag;

    % Normal distance of vector (x1 - xc)
    distn = 2.0*((x1 - xc)*rux + (y1 - yc)*ruy);

    % Ghost cell center coordinates
    geoel(1,ncx+2,j+1) = xc + distn*rux;
    geoel(2,ncx+2,j+1) = yc + distn*ruy;

    % Ghost cell area
    geoel(3,ncx+2,j+1) = geoel(3,ncx+1,j+1);
end

% Ghost cell corner geometry
% South-west corner
geoel(1,1,1) = 0.5*(geoel(1,1,2) + geoel(1,2,1));
geoel(2,1,1) = 0.5*(geoel(2,1,2) + geoel(2,2,1));
geoel(3,1,1) = geoel(3,2,2);

% South-east corner
geoel(1,ncx+2,1) = 0.5*(geoel(1,ncx+1,1) + geoel(1,ncx+2,2));
geoel(2,ncx+2,1) = 0.5*(geoel(2,ncx+1,1) + geoel(2,ncx+2,2));
geoel(3,ncx+2,1) = geoel(3,ncx+1,2);

% North-east corner
geoel(1,ncx+2,ncy+2) = 0.5*(geoel(1,ncx+1,ncy+2) + geoel(1,ncx+2,ncy+1));
geoel(2,ncx+2,ncy+2) = 0.5*(geoel(2,ncx+1,ncy+2) + geoel(2,ncx+2,ncy+1));
geoel(3,ncx+2,ncy+2) = geoel(3,ncx+1,ncy+1);

% North-west corner
geoel(1,1,ncy+2) = 0.5*(geoel(1,1,ncy+1) + geoel(1,2,ncy+2));
geoel(2,1,ncy+2) = 0.5*(geoel(2,1,ncy+1) + geoel(2,2,ncy+2));
geoel(3,1,ncy+2) = geoel(3,2,ncy+1);
end

function unkno = boundary_cond_cylinder(unkno,geoel,u_inf,v_inf)

    % Getting number of cells including ghost cells

```

```

[~,npx,npj] = size(geoel);

% No. of real cells
ncx = npx - 2;
ncy = npj - 2;

% North Ghost Cells are freestream and south wall is tengency BC
for i = 2:ncx+1
    % North
    xcn = geoel(1,i,ncy+2);
    ycn = geoel(2,i,ncy+2);

    unkno(1,i,ncy+2) = u_inf*xcn + v_inf*ycn;

    % South
    % Ghost cells share wall with real cells in North
    nxs = geoel(6,i,1);
    nys = geoel(7,i,1);

    distn = geoel(2,i,2) - geoel(2,i,1);

    u_local = unkno(2,i,2);

    if abs(nys) < 1e-12
        unkno(1,i,1) = unkno(1,i,2);
    else
        unkno(1,i,1) = unkno(1,i,2) + u_local*(nxs/nys)*distn;
    end
end

% East and west ghost cells are far field BC
for j = 2:ncy+1
    % West
    xcw = geoel(1,1,j);
    ycw = geoel(2,1,j);

    unkno(1,1,j) = u_inf*xcw + v_inf*ycw;

    % East
    xce = geoel(1,ncx+2,j);
    yce = geoel(2,ncx+2,j);

    unkno(1,ncx+2,j) = u_inf*xce + v_inf*yce;
end

end

% This function uses the phi calculations for each cell center to calculate
% the velocity components and stores them in unkno array. Then it uses
% those components along with freestream flow values to get the updated
% density storing it in the unkno array as well. Ghost cell values for the
% all are just similar to the interior cells they share walls with.

```

```

function unkno = DensityfromPhi(unkno, geoel, gamma, Mach, a_inf, rho_inf)

    % Getting number of cells including ghost cells
    [~, npx, npy] = size(geoel);

    % No. of real cells
    ncx = npx - 2;
    ncy = npy - 2;

    % Velocity from Phi only for the real cells. Does not include ghost
    % cells. Ghost cell values can be copied from the real cells.
    for j = 1:ncy
        for i = 1:ncx

            % Taking area from previously calculated function
            area = geoel(3,i,j);

            % Face averaged Phi Values
            phiE = 0.5*(unkno(1,i+1,j+1) + unkno(1,i+2,j+1));
            phiN = 0.5*(unkno(1,i+1,j+1) + unkno(1,i+1,j+2));
            phiW = 0.5*(unkno(1,i+1,j+1) + unkno(1,i,j+1));
            phiS = 0.5*(unkno(1,i+1,j+1) + unkno(1,i+1,j));

            % Extracting outwards center normal vectors
            % East
            nEx = geoel(4,i+1,j+1);
            nEy = geoel(5,i+1,j+1);

            % North
            nNx = geoel(6,i+1,j+1);
            nNy = geoel(7,i+1,j+1);

            % West
            nWx = geoel(8,i+1,j+1);
            nWy = geoel(9,i+1,j+1);

            % South
            nSx = geoel(10,i+1,j+1);
            nSy = geoel(11,i+1,j+1);

            % Velocity components u and v by multiplying phi values with
            % vector normals and dividing by the area
            % u
            unkno(2,i+1,j+1) = (phiE*nEx + phiN*nNx + phiW*nWx + phiS*nSx)/
area;

            %v
            unkno(3,i+1,j+1) = (phiE*nEy + phiN*nNy + phiW*nWy + phiS*nSy)/
area;

        end
    end

    % Density from velocity using the provided formula. This is
    % also only for the real cells.

```

```

for j = 1:ncy
    for i = 1:ncx

        u = unkno(2,i+1,j+1);
        v = unkno(3,i+1,j+1);

        % Density Calculation
        rho = rho_inf*(1 + (gamma - 1)/2 * (Mach^2 - (u^2 + v^2)/
a_inf^2))^(1/(gamma - 1));

        % Putting it in the unknown array
        unkno(4,i+1,j+1) = rho;
    end
end

% Filling in the value of the ghost cells
% North and south ghost cell boundary
for i = 1:ncx

    % North
    unkno(2,i+1,npj) = unkno(2,i+1,npj-1);
    unkno(3,i+1,npj) = unkno(3,i+1,npj-1);
    unkno(4,i+1,npj) = unkno(4,i+1,npj-1);

    % South
    unkno(2,i+1,1) = unkno(2,i+1,2);
    unkno(3,i+1,1) = unkno(3,i+1,2);
    unkno(4,i+1,1) = unkno(4,i+1,2);
end

% East and west ghost cell boundary
for j = 1:ncy

    % East
    unkno(2,npx,j+1) = unkno(2,npx-1,j+1);
    unkno(3,npx,j+1) = unkno(3,npx-1,j+1);
    unkno(4,npx,j+1) = unkno(4,npx-1,j+1);

    % West
    unkno(2,1,j+1) = unkno(2,2,j+1);
    unkno(3,1,j+1) = unkno(3,2,j+1);
    unkno(4,1,j+1) = unkno(4,2,j+1);
end

% Corner cells the velocity and other flow properties remain zero as
% the surrounding cells are ghost cells which do not depend on the
% corner cell for their property calculations
end

% This function calculates the residuals and the flux coefficients for the
% each wall of the cell along with its center. Each face contribution is
% calculated seperately by taking average of density, cell volume, and square
% face area. This is used to calculate the flux and the residuals. The code
% is based on the provided scrap code.

```

```

function [resi,coeff] = resi_calc(unkno,geoel)

% Getting number of cells including ghost cells
[~,npx,npj] = size(geoel);

% No. of real cells
ncx = npx - 2;
ncy = npj - 2;

% Initialization of residual and coefficient matrices
resi = zeros(ncx+2,ncy+2);
coeff = zeros(5, ncx+2, ncy+2);

for j = 2:ncy+1
    for i = 2:ncx+1

        % Density averages for cell face
        rhoE = 0.5*(unkno(4,i,j) + unkno(4,i+1,j));
        rhoN = 0.5*(unkno(4,i,j) + unkno(4,i,j+1));
        rhoS = 0.5*(unkno(4,i,j) + unkno(4,i,j-1));
        rhoW = 0.5*(unkno(4,i,j) + unkno(4,i-1,j));

        % Average cell volume
        volE = 0.5*(geoel(3,i,j) + geoel(3,i+1,j));
        volN = 0.5*(geoel(3,i,j) + geoel(3,i,j+1));
        volS = 0.5*(geoel(3,i,j) + geoel(3,i,j-1));
        volW = 0.5*(geoel(3,i,j) + geoel(3,i-1,j));

        % Squared face area
        area2E = geoel(4,i,j)^2 + geoel(5,i,j)^2;
        area2N = geoel(6,i,j)^2 + geoel(7,i,j)^2;
        area2S = geoel(10,i,j)^2 + geoel(11,i,j)^2;
        area2W = geoel(8,i,j)^2 + geoel(9,i,j)^2;

        % Coefficient calculations
        coeffE = rhoE*area2E/volE;
        coeffN = rhoN*area2N/volN;
        coeffS = rhoS*area2S/volS;
        coeffW = rhoW*area2W/volW;

        % Flux calculation
        FE = coeffE*(unkno(1,i+1,j) - unkno(1,i,j));
        FN = coeffN*(unkno(1,i,j+1) - unkno(1,i,j));
        FS = coeffS*(unkno(1,i,j-1) - unkno(1,i,j));
        FW = coeffW*(unkno(1,i-1,j) - unkno(1,i,j));

        % Residual calculation
        resi(i,j) = FE + FN + FS + FW;

        % Coeff matrix assignment
        coeff(1,i,j) = coeffE;
        coeff(2,i,j) = coeffN;
        coeff(3,i,j) = coeffW;
        coeff(4,i,j) = coeffS;
    end
end

```

```

        coeff(5,i,j) = -(coeffE + coeffN + coeffW + coeffS); % central
    end
end
end
% This function updates phi using Gauss Seidel method.

function [unkno,dphi,resi_n] = updatePhi(unkno,resi,coeff,omega)

    % Getting number of cells including ghost cells
    [~,npx,npj] = size(unkno);

    % No. of real cells
    ncx = npx - 2;
    ncy = npy - 2;

    % Intializing delta phi
    dphi = zeros(npx,npj);

    % Gauss-Seidel method
    for j = 2:ncy+1
        for i = 2:ncx+1

            coeffE = coeff(1,i,j); % East
            coeffN = coeff(2,i,j); % north
            coeffW = coeff(3,i,j); % West
            coeffS = coeff(4,i,j); % south
            coeffC = coeff(5,i,j); % center

            % temp storage
            dphitemp = dphi(i,j);

            % Gauss-Seidel calculation
            dphigs = (-resi(i,j) - coeffW*dphi(i-1,j) - coeffS*dphi(i,j-1) ...
                - coeffN*dphi(i,j+1) - coeffE*dphi(i+1,j))/coeffC;

            % SOR step
            dphi(i,j) = dphitemp + omega*(dphigs - dphitemp);
        end
    end

    for j = 2:ncy+1
        for i = 2:ncx+1
            unkno(1,i,j) = unkno(1,i,j) + dphi(i,j);
        end
    end

    % Residual normal
    resi_n = norm(resi(2:ncx+1,2:ncy+1),2);

end

```